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FOREIGN TECHNOLOGY DIVISION

MECHANICS OF SPACE FLIGHT

by

G. L. Grodzovskiy, D. Ye. Okhotsimskiy,
et al.

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By: G. L. Grodzovskiy, D. Ye. Okhotsimskiy, et al.

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Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

* ye initially, after vowels, and after ъ, ь; e elsewhere. When written as ё in Russian, transliterate as yё or ё. The use of diacritical marks is preferred, but such marks may be omitted when expediency dictates.

FOLLOWING ARE THE CORRESPONDING RUSSIAN AND ENGLISH DESIGNATIONS OF THE TRIGONOMETRIC FUNCTIONS

Russian	English
sin	sin
cos	cos
tg	tan
ctg	cot
sec	sec
cosec	csc
sh	sinh
ch	cosh
th	tanh
cth	coth
sch	sech
csch	csch
arc sin	$\sin^{-1}$
arc cos	$\cos^{-1}$
arc tg	$\tan^{-1}$
arc ctg	$\cot^{-1}$
arc sec	$\sec^{-1}$
arc cosec	$\csc^{-1}$
arc sh	$\sinh^{-1}$
arc ch	$\cosh^{-1}$
arc th	$\tanh^{-1}$
arc cth	$\coth^{-1}$
arc sch	sech^{-1}
arc csch	csch^{-1}
rot	curl
lg	log

MECHANICS OF SPACE FLIGHT

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Progress of space rocket technology brought to life new divisions of mechanics. The idea formed at the boundary of the 19th and 20th Centuries of application of jet engines for going into space stimulated development of mechanics of space flight (I. V. Meshcherskiy, 1897; K. E. Tsiolkovskiy, 1903; R. Goddard, 1919; G. Obert, 1923; F. A. Tsander, 1924-1925; V. Goman, 1925; R. Eno-Pel'tri, 1930; S. P. Korolev, 1934, and others). This science studies the motion of spacecraft as bodies of variable mass for the purpose of determining the conditions of the most economical use of technical means for solutions of the basic problem of flight.

Our Native land, having given to the world such scientists as I. V. Meshcherskiy and K. E. Tsiolkovskiy, is the native land of the theoretical bases of contemporary space rocket technology. The beginning of the mechanics of bodies of variable mass is embodied in the remarkable work of the Petersburg University professor I. V. Meshcherskiy "Dynamics of a point of variable mass" (1897), in which for the first time was derived the general equation of motion of a point of variable mass. In 1903 K. E. Tsiolkovskiy published in his pamphlet "Investigation of outer space by rocket instruments" a solution of the first problem of mechanics of space flight, determining the connection between final G_1 and initial G_0 weights of rocket equipment, exit velocity of the jet stream V and increase in the speed of the craft Δv during flight in a force-free field:

$$\frac{G_1}{G_0}=e^{-\frac{\Delta v}{V}}.$$

Using this formula, K. E. Tsiolkovskiy for the first time showed that a rocket can reach space speeds of flight with sufficient relative reserve of fuel $\bar{G}_{т}=(G_0-G_1)/G_0$.

The Soviet School of Mechanics of Space Flight ensured proper development of this science, necessary for solution of problems of investigating and conquering outer space. The contemporary state of development of this field of mechanics takes its beginning from the basic works of A. Yu. Ishlinskiy, A. A. Kosmodem'yanskiy and D. Ye. Okhotsimskiy (1946). Below are briefly stated the basic results attained by Soviet scientists in the field of mechanics of space flight. One can become better acquainted with these questions, and also with the works of foreign authors from the well-known monographs and survey works of A. A. Kosmodem'yanskiy (1951), I. N. Sadovskiy (1959), F. R. Gantmakher and L. M. Levin (1959), L. I. Sedov (1960), G. N. Duboshin and D. Ye. Okhotsimskiy (1963, 1965), Ye. V. Tarasov (1963), I. V. Ostoslavskiy (1963), G. L. Grodzovskiy, Yu. N. Ivanov and V. V. Tokarev (1963-1966), K. B. Alekseyev and G. G. Bebenin (1964), G. V. Korenev (1964), V. M. Ponomarev (1965), V. V. Beletskiy (1965), V. A. Sarychev (1965), V. A. Yegorov (1965), R. F. Appazov, S. S. Lavrov and V. P. Mishin (1966) and others.

§ 1. Optimization of the Motion of the Center of Masses of a Spacecraft. General Questions of Designing of Orbits

1.1. Equations of the variational problem. Optimization of the motion of the center of masses of a rocket is one of the basic problems of mechanics of space flight. In this connection was developed the division of mechanics of space flight, examining in total optimum relationships between weight components of a rocket, taking into account the weight of the basic elements of the propulsion system, optimum control of the propulsion system and optimum trajectories of space flight.

In the mechanics of space flight the problem of finding conditions of delivery of maximum payload G_π is separated by virtue of its determining influence on the ideology of arrangement and control of spacecraft. With this aspect is connected formulation of the problems in the plan of optimization of trajectory of motion, controlling parameters and weight components of the propulsion system.

The general formulation of the variational problem shown is that it is required to fulfill an assigned dynamic maneuver of flight from a fixed point 0 of phase space of coordinates — speeds $(\mathbf{r}_0, \mathbf{v}_0)$ to fixed point 1 $(\mathbf{r}_1, \mathbf{v}_1)$ in a fixed time T with maximum payload G_π at assigned initial weight of craft G_0. The differential connections of this variational problem with the corresponding boundary conditions can be recorded in the form

$$\left.\begin{array}{lll} \dot{G}=-gq, & G(0)=G_0, & G(T)=G_\pi+G_\varkappa, \\ \dot{\mathbf{r}}=\mathbf{v}, & \mathbf{r}(0)=\mathbf{r}_0, & \mathbf{r}(T)=\mathbf{r}_1, \\ \dot{\mathbf{v}}=g\frac{P\mathbf{e}}{G}+\mathbf{R}+\mathbf{F}, & \mathbf{v}(0)=\mathbf{v}_0, & \mathbf{v}(T)=\mathbf{v}_1. \end{array}\right\} \tag{1.1}$$

Here $\mathbf{r}$, $\mathbf{v}$, G are a set of phase coordinates ($\mathbf{r}$ and $\mathbf{v}$ are the radius vector and speed of center of masses of the craft, G is the current weight of the craft), q, P, $\mathbf{e}$ are the set of controlling functions and controlling parameters (q is the mass flow rate of the working substance, and P and $\mathbf{e}$ are the magnitude and unit vector of direction of thrust). As for the remaining designations, $\mathbf{R}=\mathbf{R}(r, t)$ is the g-vector from gravitational forces, $\mathbf{F}$ is the g-vector from other external forces (for example, from the force of aerodynamic drag), g is the coefficient of proportionality between mass and weight, and $G_\varkappa$ is the weight of the propulsion system.

In recording (1.1) the craft is considered to consist of payload G_π, reserve of working substance

$$G_p = \int_0^T gq\,dt$$

and propulsion system $G_\varkappa$. The weight formula of such a craft has the form

$$G = G_\pi + G_p + G_\varkappa. \tag{1.2}$$

The other weight components (for example, the weight of structural elements, the weight of tanks for working substance G_β and so forth) conditionally pertain to payload. According to necessity these components are also considered in the variational formulation.

The dimension of phase space describing the state of the craft can be increased with complication of the problem. To phase coordinates r, v, G can be added new coordinates, for example, t_μ – the current time of work of the propulsion system or G_β and $G_\varkappa$ – for problems of optimum discharge of tanks and engine. Then system (1.1) is supplemented by differential equations describing the change of these phase coordinates; in the notted examples these equations are

$$\dot{t}_\mu = \delta, \quad \dot{G}_\beta = -gq_\beta, \quad \dot{G}_\varkappa = -gq_\varkappa. \tag{1.3}$$

New controlling functions appear: $\delta(t) = 1$ or 0 is the function of switching the engine on or off, $q_\beta(t) > 0$, $q_\varkappa(t) > 0$ is the function responsible for discharging the tanks and sections of the engine.

System (1.1) is represented in the form of ordinary differential first order equations solved relative to derivatives. This permits formulating a variational problem as a Mayer problem and reducing it to a boundary value problem for a system of ordinary differential equations with finite relationships for controlling functions (for greater detail see the book of G. L. Grodzovskiy, Yu. N. Ivanov and V. V. Tokarev, 1966).

The initial problem is formulated thus: for system (1.1) to determine controlling functions and controlling parameters q, $\boldsymbol{P}$, $\boldsymbol{e}$, $G_{\varkappa}$, ensuring fulfillment of boundary conditions and delivering maximum payload $G_{\pi} = G(T) - G_{\varkappa}$.

In the described formulation is assigned a dynamic maneuver and initial weight and maximum payload is sought. Instead of this are examined also equivalent formulations: dynamic maneuver and useful weight are assigned and minimum initial weight is determined; useful and initial weights are assigned and minimum time of fulfillment of maneuver or the outer limit of some phase coordinate is found. The economy of fulfillment of the maneuver can be also characterized by another criterion of optimumness, for example, minimum cost of fulfillment of maneuver; such an approach is in the initial stage of investigation.

1.2. Basic characteristics of propulsion systems. The formulation shown in p. 1.1 of the problem of optimization is organically connected with the characteristics of propulsion systems. The given variational formulation should be specified for every type of propulsion system.

First, it is necessary to indicate functional expressions of thrust P and flow rate q through independent controls $\boldsymbol{u} = (u_1, \ldots, u_n)$ and parameters $\boldsymbol{w} = (w_1, \ldots, w_m)$ of the propulsion system (regulating engine performance):

$$\left.\begin{array}{ll} P = P(\boldsymbol{u}, \boldsymbol{w}), & q = q(\boldsymbol{u}, \boldsymbol{w}), \\ \boldsymbol{u}(t) \in U, & \boldsymbol{w} = \mathrm{const} \end{array}\right\} \qquad (1.4)$$

(U is the permissible region of control; into the number of parameters $\boldsymbol{w}$ must enter limiting values of controls $\boldsymbol{u}$).

Secondly, it is necessary to determine the weight of the propulsion system $G_{\varkappa}$ as a function of parameters $\boldsymbol{w}$ (the weight formula of the engine):

$$G_{\varkappa} = G_{\varkappa}(\boldsymbol{w}). \qquad (1.5)$$

The characteristics (1.4)-(1.5) will be defined as basic. According to complication problems can require additional information about engine properties, for example, about resource (p. 3.3) and about intensity of flow of rejections (p. 4.3).

Propulsion systems are split up into three large categories, depending upon the main limitation on the regulating characteristic (1.4) caused by the nature of physical processes in the engine. The main limitation from the point of view of mechanics of flight is characterized by the fact that the optimum operating mode of the engine, as a rule, corresponds to the approach to this limitation. Such limitations are limitation of the exit velocity of the jet stream, limitation of power and limitation of the thrust of the propulsion system.

To engines of limited exit velocity (§ 2) belong all thermal jet engines, exit velocity for which does not exceed the limit depending on the maximum temperature of the walls of the combustion chamber of heat exchanger. The weight of an engine of limited exit velocity depends on maximum thrust (for example, for a liquid-fuel rocket engine). Characteristics (1.4)-(1.5) for such engines are recorded in the form $(u = (P, V),\ w = (P_{max}, V_{max}))$:

$$P = P, \quad q = \frac{P}{V}, \quad G_u = \gamma P_{max} \tag{1.6}$$
$$(0 < P(t) < P_{max}, \quad 0 < V(t) < V_{max}).$$

To engines of limited power (pp. 3.1-3.3) belong systems consisting of a source of power and a rocket propelling agent transforming energy produced by the source into kinetic energy of directed motion of the jet stream. The presence of a separate source of limited power determines the basic properties and name of the examined category of propulsion systems. The regulating characteristic (1.4) and weight formula (1.5) for an engine of limited speed looks thus:

$$P = \sqrt{2Nq}, \quad q = q, \quad G_u = aN_{max} \div \gamma P_{max} \tag{1.7}$$
$$(0 < N(t) < N_{max}, \quad 0 < q(t) < q_{max}, \quad P_{max} = \sqrt{2N_{max} q_{max}}).$$

To engines with limited thrust (p. 3.4) belong sail systems, for example, solar or isotope sail, the value of thrust of which is limited by the maximum area of the sail S_{max}:

$$P < P_{max} = kS_{max}, \quad G_u = \gamma P_{max}. \tag{1.8}$$

A more detailed description of the characteristics of space propulsion systems from positions of mechanics of flight is given in a book of G. L. Grodzovskiy, Yu. N. Ivanov, and V. V. Tokarev (1966).

<u>1.3. Limitations in the designing of orbits</u>. A complex of requirements is imposed on space flight on the whole by a number of essential limitations to selection of optimum, in the sense of the problem (1.1), orbits of flight (see the survey of G. N. Duboshin and D. Ye. Okhotsimskiy, 1965).

During initial consideration is essentially investigated the whole totality of orbits ensuring, at least in principle, solution of the basic problem of flight. For example, during designing of flight to the moon it is important to present all possible trajectories, to know how to determine initial conditions necessary for realization of those or other trajectories, to determine kinematic and dynamic characteristics of orbits (time of flight, speed of encounter with the moon, necessary initial energy, condition of observation from assigned points of earth's surface and others).

During the analysis it is possible to remove orbits which better satisfy those or other requirements both with respect to efficiency of solution of the basic problem of flight and with respect to simplicity and economy of realization. These requirements for the most part are contradictory, and the final solution is most frequently the result of compromise and calculation of available real technical possibilities.

One of the most important orbit requirements is power economy of launching and orbit achievement. Two basic methods of achieving space orbits are used: continuous active section and launch from satellite orbit. The first method is technically simpler; however, in certain cases its realization causes difficulty. The fact is that selection of a launch site is inevitably limited, and for acceleration of a spacecraft of an assigned purpose it can be necessary to use trajectories steeply inclined to the local horizon. This causes growth of losses overcoming gravity and lowering of spacecraft weight. The method of continuous acceleration limits the range of directions of speed at the beginning of motion along the orbit, making more desirable not only orbits with the lowest possible initial speed, but also orbits with the least possible slope of the velocity vector to the local horizon at the end of the active section.

The method of launching from satellite orbit is free from power limitations on direction of acceleration. Any direction of velocity vector is obtained by the proper selection the time of launching to the intermediate orbit of the satellite (what gives aiming with respect to azimuth by turn of intermediate orbit together with earth in its daily motion) and selection of time of launching from satellite orbit (what gives aiming by angle of place at the expense of the fact that departure from satellite orbit occurs in a place where motion along the satellite orbit has the required direction). Acceleration of a spacecraft both during orbit achievement and during escape from it occurs at minimum angles of inclination to the local horizon and ensures maximum use of the power potential of the carrier rocket. Mastery by Soviet scientists and engineers of a method of launching spacecraft from satellite orbit is an outstanding technical achievement. Use of such a method of orbit achievement limits only the value of initial speed along the orbit, allowing orbit achievement of spacecraft of great weight, sharply expanding the range of possible orbits and facilitating the conditions for their expedient selection.

During the use of the method of orbit achievement with exit to intermediate satellite orbit, the most desirable space orbits are those with the lowest initial speed. During acceleration with continuous active section it is important that both speed and angle to the local horizon be as low as possible.

Another complex of questions connected with designing of orbits is investigation of the necessary accuracy of realization of the selected nominal orbit and selection of a correction method. In those cases in which flight is carried out without correction of trajectory on the way, the problem consists in exposure of the deviation domain of parameters at the end of the acceleration phase, so that the basic problem of flight could be solved if the deviation do not exceed the bounds of the shown region. For example, if the goal of the flight is to reach the moon, then deviations of parameters of removal are looked for with which the orbits pass through the moon and, this means the moon is reached. Naturally, the less constrained the limitations on the region of scattering of parameters of orbit achievement, the simpler realization of flight, the less requirement for accuracy of equipment ensuring orbit achievement, the lower the weight of this equipment and the higher its reliability. Therefore, it is desirable to select orbits of space flight which allow the greatest deviations of parameters of orbit achievement. This requirement can be and usually is in contradiction with optimum energy of orbit, and this situation is characteristic in questions of designing orbits.

It can turn out that the permissible region of initial deviations is excessively small and cannot be realized by existing technical means. Furthermore, knowledge the constants of celestial mechanics (such as solar parallax or elements of planet orbits) can be insufficient, so that even ideal fulfillment of conditions of orbit achievement does not guarantee achievement of the goal of space flight. In these cases should be used correction of orbit on the way — correction of parameters of motion which can be fulfilled by communication of pulses of the proper magnitude and direction in certain places of the orbit. The orbit can be corrected both once during the period of flight and several times.

Correction of orbit requires presence on board of a correcting propulsion system and a reserve of fuel. The value of additional weight which must be taken on board a spacecraft in connection with correction of orbit depends on the value of the correcting impulse on the value of total impulse in case of repeated correction. The value of correcting impulse depends on scattering of motion parameters at the end of the acceleration phase and will be greater, the greater the region of scattering. Furthermore, the value of impuse necessary for correction of orbit depends on the place on the orbit where this correction is realized. For example, if correction is produced too close to the target, then for this can be required a very great change of speed and a large correcting pulse, and consequently also considerable additional weight on board the spacecraft.

During selection of orbits preference is given to orbits which allow the simplest and most economical correction possible. Simultaneously appears the problem of optimization of correction, i.e., such selection of orbit and such selection of correction points on it that performance of correction requires minimum total impulse and minimum additional weight on board the spacecraft.

Solution of the problem of correction is connected with the need for exact determination of actual parameters of motion during flight, calculation of deviations of parameters of motion from the nominal values and calculation of necessary parameters of correction.

Determination of orbit parameters is a classical problem of celestial mechanics. However, its solution for spacecraft is connected with fulfillment of a number of specific requirements. For example, it is frequently necessary to determine orbit parameters as fast as possible. Therefore, the algorithm of calculation, which usually contains an iterative process, should be very economical and ensure both a small number of iterations and a short time of fulfillment of every iteration. Of the algorithm of calculations is required also high reliability and dependability, guaranteeing convergence of process even with insufficiently successful selection of initial approximation.

Accuracy of determination of actual orbit parameters depends on composition and accuracy of measured parameters, and also on location of the measured interval of the orbit and on its extent. At assigned composition and accuracy of measurements, as a rule, parameters of actual motion can be determined accurately (and orbits can be corrected more reliably and accurately) the larger the section on which the orbit is measured. However, unnecessary tightening is not rational, since it can lead to correction which is too late and an excessively high value of correcting impulse.

Early correction can be more economical; however, insufficient accuracy of determination of orbit parameters by the moment of its fulfillment can lead to insufficient accuracy of correction and to the necessity of its repeated fulfillment.

The given considerations illustrate the complexity and contradictory nature of problems connected with the designing of the system of measurements and correction of orbit, i.e., the designing of the flight control system. Optimum solution of the flight control problem consists in creation of a system ensuring solution of the basic problem of flight the most simply and reliably and the most economically with respect to the weights on board. Therefore, during selection it is expedient to prefer those orbits for which it is possible to carry out the most optimum control of flight.

Designing of orbits boils down to exposure and calculation of a number of contradictory orbit requirements, part of which is briefly shown above, to overall analysis and selection of an orbit which satisfies requirements to a maximum extent. Comprehensive analysis of flight requires calculation of a large number of variants. At the same time requirements for accuracy of calculations on the initial stage of design are usually not too high. Therefore, a reasonable solution is development and use of different methods allowing simply, economically and visually, although with limited accuracy, analyzing orbits with respect to satisfaction of requirements placed on them and looking for compromise variants giving the best solution of the problem on the whole.

A further stage of designing for selected variants requires definitized calculations considering all necessary factors affecting spacecraft flight. Such calculations are usually conducted by methods of numerical integration with use of the most exact constants and have as their goal the obtaining of exact values of parameters of flight and orbit achievement. Since definitized calculations are frequently very labor-consuming, the problem of development of effective methods of calculation is here not less acute than with respect to calculations for the stage of preliminary designing. An effective method of definitized calculation should combine necessary accuracy with speed of calculations. Therefore, creation of methods requires maximum use of knowledge of the orbit. For example, spacecraft motion with respect to the earth inside its sphere of action is closely to motion along a conic section with its focus at the center of earth. Motion outside the sphere of action of earth is close to heliocentric motion in an unperturbed orbit, etc. Calculation of these circumstances opens the path to improvement of the method of definitized calculations. Of course, other paths are also possible.

Methods of investigation of orbits are essentially determined by character of flight. It is possible to distinguish multiturn orbits and orbits with small angular distance. To orbits of the first type belong orbits of satellites of the earth, moon, planets, accomplishing during the time of its existence a large number of turns. Investigation and designing of such orbits is connected with use of the methods which allowing revealing the picture of evolution of the parameters of an osculating orbit with the passage of time under the influence of perturbing factors, such as the eccentricity of the field of gravitation, the influence of light pressure, etc. The problem of calculation of the process of evolution can be considered a problem of nonlinear oscillations, and wide application of different methods of averaging and technology of construction of asymptotic solutions can ensure creation of simple and effective methods both for preliminary and for definitized calculation.

Orbits with a small angular distance are, for example, orbits of flights from the earth to the moon and from earth to Mars, Venus or other planets. Orbits of such flights constitute in the first approximation arcs of conic sections, and questions of evolution do not come up here.

Approximation methods are created either without taking disturbances into account or taking them into account in a rather rough form. Thus, the orbit of flight to Mars can be considered to consist of three pieces of conic sections: undisturbed geocentric motion in the spere of action of earth, undisturbed heliocentric motion outside with its focus at the center of Mars, when motion occurs inside the sphere of its action.

Such are the basic considerations about the designing of orbits spacecraft.

Questions of concrete plotting of orbits are investigated in detail and are expounded in "Course of celestial mechanics," of M. F. Subbotin (1949, 1963), in works of V. A. Yegorov (1957, 1965), M. L. Lidov (1961, 1964), P. Ye. El'yasberg (1963), M. S. Yarov-Yarovy (1963), S. S. Tokmalayeva (1963), G. N. Duboshin (1963), D. Ye. Okhotsimskiy (1964), N. M. Teslenko (1964), A. I. Lur'ye (1965), I. Kh. Segal (1966), S. V. Petukhov (1966) and others.

§ 2. Mechanics of Space Flight with Engines of Limited Exit Velocity

2.1. General variational problem. Consideration of the variational problem of mechanics of flight (1.1) with engines of limited exit velocity of the jet stream (1.6) showed that taking into account the influence of the specific gravity of the propulsion system $\gamma = G_{\text{к}}/P_{\max}$ and the specific gravity of construction $\beta = G_{\text{б}}/G_{\mu 0}$ the complete variational problem is divided into a dynamic problem and a weight problem (G. L. Grodzovskiy, 1966-1967). The dynamic (trajectory) problem boils down to the well-known problem of rocket dynamics of optimum motion with an ideal weightless engine of limited thrust determining the most accessible final weight of the craft:

$$\max \frac{G(T)}{G_0} = \frac{G_1}{G_0}(V_{\max}, P_{\max}, r_0, \mathbf{r}_0, r_1, \mathbf{r}_1, T). \tag{2.1}$$

The algorithm of transition to solution of the complete variational problem, taking into consideration the weight of the propulsion system $G_\varkappa$ (depending on maximum thrust) and the weight of construction G_β (depending on the reserve of the working medium) is shown:

$$G_\varkappa = \max_{P_{\max}} [(1+\beta) G_1(P_{\max}) - \gamma P_{\max} - \beta G_0]. \tag{2.2}$$

Examples of solution of model problems about set of maximum energy during vertical climb and about optimum vertical landing in a constant plane-parallel gravitational field, about landing from circular orbit of a satellite and about set of hyperbolic speed during launch from circular orbit of a satellite showed that, in spite of small values of specific gravity of an engine of limited exit velocity, calculation of the weight of the propulsion system essentially affects parameters of optimum motion of a body of variable mass and leads to an extreme problem of determination of engine weight (maximum thrust) ensuring maximum deliverable payload.

2.2. The dynamic part of the problem. In connection with the separation shown in p. 2.1 of the complete variational problem into weight and dynamic parts, of fundamental importance are solutions of the problem of rocket dynamics of optimum motion with an ideal weightless engine of limited thrust $P(t) \leqslant P_{\max}$, ensuring minimum total increase in characteristic speed. The first works on the problem of optimization in rocket dynamics belong to 1946. Then A. Yu. Ishlinskiy showed that the condition of constancy of the speed of the jet stream is equivalent to the hypothesis about the fact that during rejecting of the jet stream is liberated kinetic energy proportional to the expended mass q; A. A. Kosmodem'yanskiy and D. Ye. Okhotsimskiy was investigated in detail the problem of optimum ascent of the rocket along the vertical to maximum altitude. These investigations were further developed in the works of V. V. Beletskiy (1956), V. A. Yegorov (1958), V. K. Isayev, A. I. Kur'yanov and V. V. Sonin (1964) and others. Essential was the solution published in 1957 by D. Ye. Okhotsimskiy and T. M. Eneyev (and independently of them by D. F. Louden and B. D. Frayd) of the problem of optimum girding of a satellite into circular orbit. An important result was obtained about the fact that along the optimum trajectory the tangent of the angle of the direction of thrust is a linear-fractional function of time

$$\operatorname{tg}\varphi=\frac{at+b}{ct+d}. \tag{2.3}$$

In a work of Yu. A. Gorelov (1960) were determined the conditions fulfillment of which ensures extreme motion of a rocket along a curvilinear trajectory. Composition of optimum control in the problem of rocket dynamics of motion with an ideal weightless engine of limited thrust in a plane-parallel gravitational field was investigated in detail in works of V. K. Isayev (1961-1962). He showed the effectiveness of application of the principle of maximum of L. S. Pontryagin (1961) in the solution of complex problems of rocket dynamics. The method of L. S. Pontryagin conquered the special popularity in recent years with which is connected the great progress made in the whole world in the solution of practical problems of rocket dynamics with complex limitations.

Subsequently the introduction of an extremely successful model of a uniform central field permitted solving the problem of optimum control of a point of variable mass in a central gravitational field in the presence of limitation on jet thrust (G. Ye. Kuzmak, V. K. Isayev and B. Kh. Davidson, 1963). Another important problem about the turning of the plane of the orbit of a satellite is examined in detail in works of V. F. Illurionov and L. M. Shkadov (1962), Yu. M. Kopnin (1965, 1967), Yu. N. Ivanov and Yu. V. Shalayev (1965).

A work of D. Ye. Okhotsimskiy and T. M. Eneyev (1957) also initiated investigation of the optimum problems of rocket dynamics of multistage systems. These problems were successfully developed further in works of K. A. Pobedonostsev (1958), O. F. Makarov (1962), Yu. V. Kozhevnikov (1963, 1965), V. A. Kosmodem'yanskiy (1964), V. A. Troitskiy (1965), A. A. Bolonkin (1965) and others.

2.3. Pulse flights. The influence of the weight of the propulsion system is immaterially for maneuvers during the fulfillment of which the time of work of the engine is much shorter than the time of fulfillment of the maneuver. For such maneuvers application of jet thrust can be considered pulse application.

Optimum trajectories with many pulses were investigated by V. I. Charnym (1963), who strictly proved that optimum multipulse flight consists of arcs of conic sections touching in apsidal points. Two-pulse optimum flight between orbits with small slope and eccentricities was studied by V. S. Novoselovey (1963) and optimum coplanar flights between orbits by S. N. Kirpichnikov (1964). Conditions of optimum impulse transition of a spacecraft, braked in the atmosphere of a planet to the orbit of an artificial satellite were analyzed in detail by V. A. Il'in (1963). Later V. A. Il'in (1964, 1967) and V. S. Vozhdayev (1967) examined the problem of determination of optimum trajectory of flight between coplanar circular orbits with the use of the method of spheres of action and obtained simpler algebriac relationships between the eccentricities and focal parameters of one- and two-pulse flights. One more interesting investigation of V. A. Il'in (1967) is dedicated to the approximate solution of the problem of synthesis of trajectory of close circling of the moon with reentry of the atmosphere of earth. In this investigation is successfully used replacement of spacecraft motion in the sphere of action of the moon by the "unfolding pulse" of the field of gravitation of the moon.

A complex of problems on optimum pulse flights between orbits located in a small environment of the base circular orbit, is studied in detail in works of G. Ye. Kuzmak (1965, 1967) and N. I. Lavrenko (1965, 1967). Analysis of this case is interesting for two reasons: first, small deformations of orbits in the environment of the base circular orbit lead to radical changes of parameters of pulse circuits of flights and, secondly, the region of applicability of approximate solutions built by such a method is sufficient for investigation of a broad class of circumplanetary maneuvers. Modified parameters in investigations were moments of application of pulses, their components and the number of pulses ensuring minimum total increase in characteristic speed. G. Ye. Kuzmak (1965) by this method solved the two-dimensional problem of optimum flight from the initial orbit to a point lying in the plane of the orbit, and the problem of flights between arbitrary coplanar orbits. In 1965 G. Ye. Kuzmak and N. I. Lavrenko brought also solution of the problem of flights from initial orbit to a point lying outside the plane of the orbit, and in 1967 the same method they examined optimum flights between noncoplanar orbits.

Optimum three-pulse turn of the plane of a circular orbit was investigated by L. V. Zakoteyeva and V. V. Polyachenko (1965). Optimum orbits of one- and two-pulse flights between points moving along one orbit were analyzed in detail by S. N. Kirpichnikov (1966). Pulse flights between different orbits were examined in works of S. V. Dubovskiy (1964), V. S. Novoselov (1965), V. V. Ivashkin (1966) and others.

§ 3. Mechanics of Space Flights with Engines of Limited Power

The first works on the problem of optimization in problems of mechanics of flight with an engine of limited power belong to 1959-1961 (J. Irving and E. Blum, 1959; G. L. Grodzovskiy, Yu. N. Ivanov and V. V. Tokarev, 1961). In them were considered the chief features of the characteristics of such engines: the limitedness of the power of the jet stream and the dependence of engine weight on maximum power. The fact of separation of the initial problem into weight and dynamic parts was established. Basic properties of optimum solutions were revealed: the presence of the best distribution of starting the weight between engine and the working substance and the advantage of change of the value of thrust in the process of flight.

Further investigations developed in two directions. The first is inclusion of the real characteristics of the engine in the formulation of problems of optimization and consideration of modified circuits of an engine of limited power. The second is solution of problems of optimization for different maneuvers in gravitational fields close to real ones. These investigations are accompanied by the development of various kinds of procedures which allow using variational methods and the construction of approximate and numerical methods of solution of variational and boundary value problems.

3.1. The ideal engine is characterized by absence of losses of working substance and power, its regulation is governed only by limitation on power, and the weight of the propulsion system linearly depends on maximum power (see 1.7) at $\gamma = 0$, $q_{max} = \infty$). Study of the ideal case is interesting because it opens maximum possibilities of engines of a given class.

Here are examined possible variants of control of engine weight: constant engine weight (J. Irving and E. Blum, 1959; G. L. Grodzovskiy, Yu. N. Ivanov and V. V. Tokarev, 1961), stepwise and continuously variable engine weight (G. L. Grodzovskiy, 1961; Yu. N. Ivanov, 1962, 1964), use of dropped sections of the engine as working substance (V. V. Tokarev, 1963, 1965). In the first case is performed generalization for nonlinear dependence of engine weight on maximum power (G. L. Grodzovskiy, 1965).

Let us describe formulation of the variational problem on an example of the maximum case – continuous discharge of infinitesimal sections of the engine ($\dot{G}_\varkappa = -gq_\varkappa$, $q_\varkappa(t) > 0$) with their partial transformation into working substance ($0 < \varkappa(t) < \varkappa_{max} = \text{const} < 1$). Part of the flow rate $q_\varkappa$, equal to $(1 - \varkappa)\, q_\varkappa$ is not used and abandons the craft with zero speed; the remaining part $\varkappa q_\varkappa$, transformed into working substance, is sent to the propelling agent. The total flow rate through the propelling agent q will be composed of flow rate $\varkappa q_\varkappa$ and the flow rate of reserve of working substance q_μ. Having demanded $q_\varkappa(t) \equiv 0$, we will obtain the case of constant engine weight; at $\varkappa_{max} = 0$ we will have the case of passive discharge of engine sections.

Let us connect to the system (1.1) the third of the equations (1.3) and depict it taking into account engine performance (1.7) at $\gamma = 0$, $q_{max} = \infty$):

$$\left.\begin{array}{lll} \dot{G}_\sigma = -q_\mu, & G_\sigma(0) + G_\varkappa(0) = 1, & G_\sigma(T) = \max, \\ \dot{G}_\varkappa = -q_\varkappa, & & G_\varkappa(T) \geqslant 0, \\ \dot{r} = v, & r(0) = r_0, & r(T) = r_1, \\ \dot{v} = \dfrac{\sqrt{2\frac{g}{\alpha} G_\varkappa N (q_\mu + \varkappa q_\varkappa)}}{G_\sigma + G_\varkappa} e + R, & r(0) = r_0, & r(T) = r_1 \end{array}\right\} \quad (3.1)$$

$$\left(\begin{array}{c} |e(t)| = 1, \quad 0 < N(t) < 1, \quad 0 \leqslant \varkappa(t) \leqslant \varkappa_{\max}, \\ 0 < q_\mu(t) < \infty, \quad 0 < q_\varkappa(t) < \infty \end{array} \right).$$

Here all weights are referred to the initial weight of the craft, flow rates to the initial mass, and power to the highest possible power at a given moment. The symbol G_σ designates total weight $G_\sigma(t) = G_\pi + G_\mu(t)$.

It is necessary to select the optimum initial value of weight $G_\varkappa$ and to construct optimum programs for controls $e(t)$, $N(t)$, $\varkappa(t)$, $q_\mu(t)$ and $q_\varkappa(t)$. Design parameters – the specific weight of the engine α and the maximum coefficient $\varkappa_{max}$ of transformation of the material of the engine into working substance – are assigned; dynamic maneuver $\{r_0, v_0; r_1, v_1; T\}$ is fixed. The maximized functional is the final value of phase coordinate G_σ coinciding in definition with useful weight G_π.

Analysis of the structure of optimum control made on the basis of the principle of maximum of L. S. Pontryagin (1961) permits in all cases ($1° - q_\varkappa(t) \equiv 0$; $2° - q_\varkappa(t) \geqslant 0$, $\varkappa_{\max} = 0$; $3° - q_\varkappa(t) \geqslant 0$, $1 > \varkappa_{\max} > 0$) breaking up the initial variational problem (3.1) into weight and dynamic parts.

The weight part of the problem is solved to the end analytically. The optimum partition of initial weight between the engine $G_{\varkappa 0}$, the reserve of working substance $G_{\mu 0}$, and the payload G_π is determined, and the connection between payload G_π and the functional

$$\Phi = \frac{\alpha}{2g} \int_0^T a^2 \, dt,$$

characterizing the trajectory is found.

For the case of constant engine weight $(q_{\varkappa}(t)=0)$

$$G_{\varkappa}=\sqrt{G_{\pi}}-G_{\pi}, \quad G_{\pi}=(1-\sqrt{\Phi})^{2} \quad (0<\Phi<1); \tag{3.2}$$

for the case of passive discharge $(q_{\varkappa}(t)>0,\ \varkappa_{max}=0)$

$$\left.\begin{array}{l} G_{\varkappa 0}=\begin{cases}\sqrt{G_{\pi}}-G_{\pi} & \text{at } \frac{1}{4}<G_{\pi}<1,\\ \frac{1}{4} & \text{at } 0<G_{\pi}<1;\end{cases}\\ G_{\pi}=\begin{cases}(1-\sqrt{\Phi})^{2} & \text{at } 0<\Phi<\frac{1}{4},\\ \frac{1}{4}\exp(1-4\Phi) & \text{at } \frac{1}{4}<\Phi<\infty;\end{cases}\end{array}\right\} \tag{3.3}$$

for the case of active discharge $(q_{\varkappa}(t)>0,\ 1>\varkappa_{max}>0)$

$$\left.\begin{array}{l} G_{\varkappa 0}=\begin{cases}K-G_{\pi} & \text{at } C<G_{\pi}<1,\quad 0<\varkappa_{max}<\frac{1}{2},\\ & \text{and } 0<G_{\pi}<1,\quad \frac{1}{2}<\varkappa_{max}<1,\\ \frac{1}{4(1-\varkappa_{max})} & \text{at } 0<G_{\pi}<C,\quad 0<\varkappa_{max}<\frac{1}{2};\end{cases}\\ \Phi=\begin{cases}(1-\varkappa_{max})\left(1-\frac{G_{\pi}}{K}\right)-\varkappa_{max}\ln\frac{G_{\pi}}{K}+ & \\ \qquad +G_{\pi}-K & \text{at } C<G_{\pi}<1,\ 0<\varkappa_{max}<\frac{1}{2}\\ & \text{and } 0<G_{\pi}<1,\ \frac{1}{2}<\varkappa_{max}<1,\\ C-\varkappa_{max}\ln C-\frac{1}{4(1-\varkappa_{max})}\ln\frac{G_{\pi}}{C} & \text{at } 0<G_{\pi}<C,\ 0<\varkappa_{max}<\frac{1}{2}\end{cases}\end{array}\right\} \tag{3.4}$$

$$\left(K=\frac{1}{2}\varkappa_{max}+\sqrt{\frac{1}{4}\varkappa_{max}^{2}+(1-\varkappa_{max})G_{\pi}},\quad C=\frac{1-2\varkappa_{max}}{4(1-\varkappa_{max})}\right).$$

Reserve of working substance is calculated as $G_{\mu 0}=1-G_{\pi}-G_{\varkappa 0}$. The current values of combining weight G(t) and weight of engine $G_{\varkappa}(t)$ are expressed through the current value of the integral $\Phi(t)$.

Analogous procedure is made in the case of instantaneous discharge of final sections of the engine.

The dynamic (or trajectory) part of the problem boils down to minimization of the integral from the square of rocket acceleration with differential connections — equations of motion

$$\left.\begin{array}{lll} \dot{J} = a^2, & J(0) = 0, & J(T) = \min, \\ \dot{r} = v, & r(0) = r_0, & r(T) = r_1, \\ \dot{v} = ae + R, & v(0) = v_0, & v(T) = v_1 \end{array}\right\} \qquad (3.5)$$

$$(a(t) > 0, \ |e(t)| = 1)$$

or, after exclusion of the latter, to minimization of the integral functional

$$J = \int_0^T [\ddot{r} - R(r, t)]^2 \, dt \qquad (3.6)$$

$$(r(0) = r_0, \quad \dot{r}(0) = v_0; \quad r(T) = r_1, \quad \dot{r}(T) = v_1).$$

An analysis was made of the properties of extremals, and solutions were analytically obtained of a number of model problems (G. L. Grodzovskiy, 1961; Yu. N. Ivanov, 1961, 1964; V. V. Tokarev, 1961, 1964; Yu. V. Shalayev, 1964; V. K. Isayev, 1962, 1964; V. V. Sonin, 1962, 1964; B. Kh. Davidson, 1964; V. V. Beletskiy and V. A. Yegorov, 1964; L. A. Lebedev and S. A. Sakovskiy, 1964). Detailed numerical solutions were obtained, and a number of approximate analytic solutions were built for problems of flight to planets of the solar system and maneuvers in the vicinity of a planet (G. L. Grodzovskiy, 1961; Yu. N. Ivanov, 1961, 1964-1965; V. V. Beletskiy, 1964-1965; V. A. Yegorov, 1964-1965; V. G. Yershov, 1965; V. K. Isayev, A. I. Kur'yanov and V. V. Sonin, 1964; S. A. Pokrovskaya, 1964; G. B. Yefimov and D. Ya. Okhotsimskiy, 1965, and others).

3.2. Unregulated engines are characterized by constancy of thrust and flow rate, only the turning off of the engine is allowed, (then thrust and flow rate are equal to zero), and no limitations are put on change of direction of thrust. This is the second extreme case of regulation.

A variational problem for unregulated engines is also divided into weight and dynamic parts. The latter, in contrast to the ideal case, contains two engine parameters – initial rocket acceleration $a_0 = g\,P/G_0$ and exit velocity V (for dimensionless flow rate $\mu = gq/G_0 = a_0/V$), but is universal for all types of unregulated engine (Yu. N. Ivanov and Yu. V. Shalayev, 1965). The weight part is a problem of minimization of a function of two variables

$$G_n = \max_{P,V}\left\{G_0 - G_x(P, V) - \frac{gP}{V} T_\mu\left(\frac{gP}{G_0}, V\right)\right\} \qquad (3.7)$$

and can be solved for every concrete type of engine $G_\varkappa(P, V)$, as only the solution of the dynamic of the problem $T_\mu(a_0, V)$.

The dynamic part boils down to a variational problem about the minimum time of work of the engine T_μ at the assigned time of motion T (or any of its equivalents):

$$\left.\begin{array}{lll} \dot{t}_\mu = \delta, & t_\mu(0) = 0, & t_\mu(T) = \min, \\ \dot{r} = v, & r(0) = r_0, & r(T) = r_1, \\ \dot{v} = \dfrac{a_0\delta}{1-\mu t_\mu} e + R, & v(0) = v_0, & v(T) = v_1 \end{array}\right\} \qquad (3.8)$$
$$(a_0,\ \mu = \text{const},\ \delta(t) = 1 \text{ or } 0,\ |e(t)| = 1).$$

Here it is required to construct optimum programs of switching on $(\delta(t) = 1)$ – turning off $(\delta(t) = 0)$ the engine and orientation of thrust vector e(t).

Although formulation of this problem is suitable for any unregulated engine, solution results are divided into two large classes (for engines of small thrust and for engines of large thrust), depending upon the range of parameters a_0 and μ for which it is obtained. The last remark touches numerical and approximate analytic solutions. In the class of such solutions pertaining to engines of small thrust is investigated approximately the same set of maneuvers as for an ideal engine of limited power (V. N. Lebedev, 1963, 1966; B. N. Rumyantsev, 1963; N. N. Moiseyev, 1966; V. V. Beletskiy and V. A. Yegorov, 1964; Yu. N. Ivanov, and Yu. V. Shalayev, 1965; Yu. M. Kopnin, 1965; R. F. Avramchenko, V. M. Bezmenov, V. A. Vinokurov and V. V. Tokarev, 1967, and others), indeed, less complete numerical results were obtained for interplanetary flights.

3.3. Real engines are characterized by the presence of losses of working substance and power and by limitations to control range and to resource; furthermore, the weight formula of the real propulsion system contains several components. In general the problem of optimization here is no longer divided into weight and dynamic parts and should be solved as an integral problem.

Besides the weight components of the craft, which were considered in the preceding account, it is possible to name still at least two: the weight of tanks for working substance $G_\beta = \beta G_{\mu 0}$ and the weight of the propelling agent $G_\gamma = \gamma P_{max}$. The variation formulation (1.1) will look thus:

$$\left.\begin{aligned} &\dot{G} = -\frac{gP^2}{2N}, \qquad G(0) = G_0, \quad G_\pi = G(T) - \beta(G_0 - G(T)) - \\ &\qquad\qquad\qquad\qquad\qquad - \alpha N_{max} - \gamma P_{max} = \max, \\ &\dot{\mathbf{r}} = \mathbf{v}, \qquad\qquad \mathbf{r}(0) = \mathbf{r}_0, \qquad \mathbf{r}(T) = \mathbf{r}_1, \\ &\dot{\mathbf{v}} = \frac{gP}{G}\mathbf{e} + \mathbf{R}, \quad \mathbf{v}(0) = \mathbf{v}_0, \qquad \mathbf{v}(T) = \mathbf{v}_1 \\ &\quad (0 < P(t) < P_{max}, \quad 0 \leqslant N(t) \leqslant N_{max}, \quad |\mathbf{e}(t)| = 1). \end{aligned}\right\} \tag{3.9}$$

A feature of this problem is that the maximized function G_π depends on limitations N_{max} and P_{max}, superimposed on controls $N(t)$ and $P(t)$ and subject to optimization. By introduction of dimensionless control $\bar{P}(t) = P(t)/P_{max}$, $\bar{N}(t) = N(t)/N_{max}$ and formal equations $\dot{P}_{max} = 0$, $\dot{N}_{max} = 0$ the problem can be reduced to the problems of optimum control without parameters, for investigation of which we will use the principle of maximum in the standard formulation (Yu. N. Ivanov, 1954).

Very acute for engines of limited power is the problem of resource, since the optimum time of work of such an engine coincides with the whole time of motion of composes a considerable part of it. A general method is developed of solution of variational problems with a fixed time of control actions, on the basis of which is solved a number of problems of optimum control of an engine with limited resource (Yu. N. Ivanov, 1963, 1965; V. A. Vinokurov, 1965).

The idea of the method is the introduction of a new phase coordinate t_μ – the current time of work of the engine – and a new controlling function $\delta(t) = 1$ or 0, responsible for the switching on $(\delta = 1)$ and off of the engine $(\delta = 0)$. The connection between them is given by the differential equation $\dot{t}_\mu = \delta$ with boundary conditions $t_\mu(0) = 0$, $t_\mu(T) < T_\mu$, where T_μ is the assigned resource of the engine. In equation (1.1) thrust P and flow rate q are replaced with $P\delta$ and $q\delta$:

$$\left.\begin{array}{lll} \dot{G} = -gq\delta, & G(0) = G_0, & G_\pi = G(T) - G_\kappa = \max, \\ \dot{t}_\mu = \delta, & t_\mu(0) = 0, & t_\mu(T) < T_\mu, \\ \dot{r} = v, & r(0) = r_0, & r(T) = r_1, \\ \dot{v} = \frac{gP\delta}{G} e + R, & v(0) = v_0, & v(T) = v_1. \end{array}\right\} \qquad (3.10)$$

Here an analysis of optimum control is given, and analytic and numerical solutions are obtained for maneuvers in force-free and central fields.

The formulation is given and a procedure is developed of solving the problem of the best approximation of the continuous law of control to piecewise-constant control with the assigned and optimum number of levels. Such a problem appears, for example, in case of application of an engine with a narrow range of regulation when the ideal program of thrust requires deep regulation.

The procedure, like the preceding one, is based on the use of relay controlling functions $\delta_i(t) = 1$ or 0. Thrust P is presented in the form

$$P(\pi_i, \delta_i) = ((\ldots(\pi_1\delta_1 + \pi_2)\delta_2 + \ldots + \pi_{s-2})\delta_{s-2} + \pi_{s-1})\delta_{s-1} + \pi_s, \qquad (3.11)$$

where π_i are constant parameters determining the altitude s of levels of thrust $P_1 = \pi_1 + \pi_2 + \ldots + \pi_s$, $P_2 = \pi_2 + \ldots + \pi_s$, ..., $P_s = \pi_s$; controls δ_i $(i = 1, \ldots, s-1)$ are independent. Having placed expression (3.11) in equations (3.9), it is possible to obtain conditions for optimum values of parameters π_i; the weight of propelling agent γP_{max} must be replaced with the sum of the weights of s variously tuned propelling agents $\gamma \Sigma P_i = \gamma \Sigma i\pi_i$.

Further investigation is conducted with the help of the principle of maximum. Analytical and numerical examples are obtained of solution of the problem of step approximation of thrust for basic maneuvers (Yu. N. Ivanov, 1964, 1966).

The features of optimum controls of engines with real regulating characteristics in the presence of additional limitations on control parameters (V. K. Isayev, 1962, 1964; V. V. Sonin, 1962, 1964; B. Kh. Davidson, 1964; A. I. Kur'yanov, 1964; Yu. N. Ivanov, 1966).

3.4. Propulsion system related to engines of limited power and limited exit velocity. A large number of investigations have been conducted on problems of optimization of mechanics of flight with engines which are modifications of two basic types, and with engines of other schemes. To them belong:

engines of limited power with batteries of energy, thermal or electrical (G. L. Grodzovskiy, 1965, 1967; B. N. Kiforenko, 1965-1967; V. V. Tokarev, 1965);

engines of limited power and limited exit velocity with accumulation of atmospheric gas utilized as working substance (V. V. Tokarev, 1965; Yu. M. Fatkin, 1965, 1967; G. L. Grodzovskiy, 1966);

solar and isotope sails (F. A. Tsander, 1924; A. N. Zhukov and V. N. Lebedev, 1964; K. G. Valeyev, 1964; G. L. Grodzovskiy, 1966);

engine with solar and isotope sources of energy (A. Ye. Ilyutovich, 1967; R. N. Ovsyannikov and L. N. Semenov, 1967).

§ 4. Additional Aspects of the Problem of Optimization in the Mechanics of Space Flight

4.1. Optimum combination of engines of various types. Solution of the problem of optimization does not end with selection of the best parameters of a given type of engine. It is still necessary to clarify what type of engine is suitable to use for execution of a given maneuver, and to determine the expediency of joint use of various types of engines on one craft.

Let us assume that on the spacecraft are two engines 1 and 2, which can work either in parallel or in series. Then the flow rate q and thrust vector Pe in equations (1.1) are replaced in parallel operating conditions with

$$q = q_1 + q_2, \quad Pe = P_1e_1 + P_2e_2, \tag{4.1}$$

and in series operating conditions with

$$q = q_1\delta + q_2(1-\delta), \quad Pe = P_1e_1\delta + P_2e_2(1-\delta), \tag{4.2}$$

where $\delta(t) = 1$, when engine 1 is on, and $\delta(t) = 0$, when engine 2 is on. Introduction of this relay controlling function permits considering the controls of engines 1 and 2 independent. The weight of engine $G_\varkappa$ in boundary conditions (1.1) is recorded in the form of the sum $G_\varkappa = G_{\varkappa1} + G_{\varkappa2}$. The regularity (1.4) and weight (1.5) characteristics of engines 1 and 2, as usual, lock the variational problem.

Cases are investigated of parallel and series operating modes of engines of limited exit velocity (large thrust) and limited power (small thrust), and examples are given of construction of the boundary of the field of application of engines of limited power for different maneuvers (Yu. N. Ivanov, 1964, 1966).

4.2 Modification of the criterion of optimumness. Usually during the formulation of variational problems is used the weight criterion of optimumness – the maximum payload at the assigned starting weight (dynamic maneuver is fixed).

If the same maneuver is to be performed repeatedly, it is expedient to replace the weight criterion with the cost criterion. The cost of fulfillment of the maneuver is composed of the cost of achievement of initial orbit and the costs of the basic components of the craft; the latter are considered proportional to the corresponding weights.

The cost of delivery of a unit of payload is minimized. The functional of the problem has such a structure:

$$S=\frac{1+s_{\varkappa}G_{\varkappa}+s_{\mu}G_{\mu0}}{1-G_{\varkappa}-G_{\mu0}}=\min, \tag{4.3}$$

where $s_{\varkappa}$ and s_{μ} are dimensionless coefficients characterizing the cost of the engine and the working substance. Minimum is looked for with connections (1.1)-(1.2). As a result optimum controls remain the same, and the optimum values of constant controlling parameters — weight relationships, etc., changed (V. V. Tokarev, 1966).

For the purpose of reducing expenditures of designing and development of a propulsion system, the problem is posed of selection of parameters of a universal engine (one or more) ensuring fulfillment of maneuvers from a certain region B

$$\mathbf{b}=(\mathbf{r}_0, \mathbf{v}_0; \mathbf{r}_1, \mathbf{v}_1; T)\in B. \tag{4.4}$$

The quality of fulfillment of every maneuver **b** is characterized by the function x(T) — maximum payload G_{π} or minimum cost S. The controlling functions of the engine $\mathbf{u}(t)$ from (1.4) are selected in such a manner that connections (1.1) are satisfied and the extremum of functional x(T) is attained:

$$\underset{\mathbf{u}(t)\in U}{extr}\; x(T)=x_1(\mathbf{b}, \mathbf{w}). \tag{4.5}$$

Engine parameters $\mathbf{w}$ besides are fixed, controls $\mathbf{u}(t)$ are selected from the permissible region U and can be different depending upon maneuver parameters **b**. The problem of construction of optimum control $\mathbf{u}(t)$ will be considered solved and dependence (4.5) known.

Maneuver parameters **b** can with frequency ν take any values from fixed region B or fixed discrete set B. The frequency $\nu(\mathbf{b})$ of repetitions of maneuver **b** is recorded through distribution function $F_{\nu}(\mathbf{b})$ with the help of the integral of Stieltjes (step distribution functions are allowed). The frequency $\nu(\mathbf{b}\in B')$ of repetition of maneuvers belonging so subdomain $B'\subset B$, is equal to

$$\nu(b \in B') = \int_{b \in B'} dF_\nu(b) \qquad \left(\int_{b \in B} dF_\nu(b) = 1\right). \tag{4.6}$$

Vector w of the engine parameters for all values of $b \in B$ can take only one value — the requirement of universality of the engine for the assigned class of maneuvers. If it is possible for every maneuver **b** to select its own system **w**(b), as in the preceding formulations, then it is possible to ensure the greatest (least) value of functional (4.5)

$$\underset{\substack{u(t) \in U \\ w \in W}}{\text{extr}}\, x(T) = \underset{w \in W}{\text{extr}}\, x_1(b,\ w) = x_1(b), \tag{4.7}$$

i.e., the ideal solution. A universal engine will ensure a value of functional (4.5) less (greater) than (4.7), with the exception of that maneuver **b** for which parameters of a universal engine **w** are optimum in the sense of condition (4.7):

$$\Delta x_1(b,\ w) = x_1(b) - x_1(b,\ w) \begin{cases} > 0, & \text{if max } x(T) \text{ is looked for,} \\ < 0, & \text{if min } x(T) \text{ is looked for,} \end{cases} \tag{4.8}$$

The effectiveness of a universal system depending upon the character of the problem can be characterized either by the functional averaged from all maneuvers $b \in B$ referred to the ideal value

$$\langle \bar{x}_1 \rangle = \frac{\int_{b \in B} x_1(b,\ w)\, dF_\nu(b)}{\int_{b \in B} x_1(b)\, dF_\nu(b)}, \tag{4.9}$$

or by the averaged loss (4.8)

$$\langle \Delta \bar{x}_1 \rangle = \int_{b \in B} \left| \frac{x_1(b) - x_1(b,\ w)}{x_1(b)} \right| dF_\nu(b). \tag{4.10}$$

If distribution function $F_\nu(b)$ is known, then the problem boils down to detecting extr$\langle \bar{x}_1 \rangle$ or min $\langle \Delta\bar{x}_1 \rangle$ for parameters **w**, i.e., to the problem on extremum of function of several variables (V. V. Tokarev, 1964, 1966). Solution of the variational problem (4.5) is considered found for any $\mathbf{b} \in \mathbf{B}$ and $\mathbf{w} \in \mathbf{W}$; otherwise the problem of universality boils down to the problem of optimum control of the distributed system (Yu. V. Kozhevnikov, 1966).

If, however, distribution $F_{\nu}(\mathbf{b})$ is now known beforehand, then the game approach is used. Distribution $F_{\nu}(\mathbf{b})$ such as would maximize loss (4.10), and parameters $\mathbf{w}$ are looked for such as would minimize this loss. In the case of a continuous game and an integrand convex with respect to $\mathbf{w}$ (4.10) the problem becomes one of detecting minimax

$$\min_{w \in W} \max_{b \in B} \left| \frac{z_1(b) - z_1(b, w)}{z_1(b)} \right|. \tag{4.11}$$

Solution of the problem of universalization in such a formulation determines engine parameters optimum in the following sense: whatever distribution $F_{\nu}(\mathbf{b})$ of recurrence rate of maneuvers is assigned, loss (4.10) will not exceed the value found (V. V. Tokarev, 1966; Yu. M. Fatkin, 1966).

After the parameters of a universal engine are selected, for every concrete maneuver is determined the optimum reserve of working substance (V. V. Tokarev and R. F. Avramchenko, 1967).

4.3. Questions of reliability in problems of optimization. During flight the propulsion system can be acted upon by accidental factors causing failures. In accordance with the character of processes leading failures, the probability of failures can depend on time, coordinates, engine parameters and craft and on the operating conditions of the engine. Therefore, calculation of reliability factor leads to change in habitual programs of optimum control and optimum parameters.

The intensity of flow of failures, λ, is assumed known as a function of time t, coordinates r, controlling functions u and parameters $\mathbf{w}$ of the engine:

$$\lambda = \lambda(t, r, u, w). \tag{4.12}$$

With the assigned probability R it is guaranteed that in every realization of maneuver the value of the criterion of quality will be not less (not more) than the calculated value (for example, the actual fuel consumption will not exceed calculation).

In the case of a monosectional engine this condition coincides with the condition of unfailing work of the engine during the time of fulfillment of maneuver

$$\int_0^T \lambda(t,\ r(t),\ u(t),\ w)\,dt < -\ln R. \tag{4.13}$$

To equations (1.1) in accordance with (4.13) is added the equation for the additional phase coordinate Λ responsible for reliability

$$\dot{\Lambda} = \lambda, \quad \Lambda(0) = 0, \quad \Lambda(T) < -\ln R, \tag{4.14}$$

after which the problem a standard problem of optimum control, which is solved by the method of L. S. Pontryagin.

When the intensity of flow of failures λ and the full time of motion T are great $(\lambda_{max}T \gg 1)$, then a monosectional propulsion system cannot ensure an acceptable level of payload with a sufficiently high probability of realization. In this case the propulsion system (or its most unreliable elements) should be divided into autonomous sections. Sections are considered equivalent; damage of every section leads to impairment of engine performance (for example, to decrease in power for engines of limited power), but does not cause cessation of work of the engine on the whole; all sections work in parallel. Regulating (1.4) and weight (1.5) characteristics of the engine will contain now still a number of working sections

$$\left.\begin{array}{ll} P = P(u,\ w,\ n), & q = q(u,\ w,\ n), \\ G_u = G_u(w,\ n_0), & u(t) \in U,\ w = \mathrm{const}. \end{array}\right\} \tag{4.14'}$$

In order to guarantee here with the assigned probability R realization of values of functional not less (not greater) than calculation values, it is necessary to construct a law of damages n(t) (nonaccidental function), which with probability R limits from below all realizations $\tilde{n}(t)$ (random functions):

$$P[\tilde{n}(t) > n(t) \text{ at } 0 < t < T] > R. \tag{4.15}$$

Is proven that at sufficiently high probability $R(1-R \ll 1)$ condition (4.15) is fulfilled, law n(t) is constructed in the following way:

$$\left.\begin{array}{l} n(t)=n_0-j \quad \text{at} \quad t_j<t<t_{j-1}, \\ \int\limits_{t_j}^{t_{j+1}} \lambda(t,\ r(t),\ u(t),\ w)\,dt=\frac{-\ln R}{n_0-j}\ (j=0,\ 1,\ \ldots,\ m;\ t_{m+1}>T) \end{array}\right\} \qquad (4.16)$$

(analog of condition (4.13)). Relationships (4.16) mean that if by instant t_j j sections of n_0 are damaged, then in the interval (t_j, t_{j+1}) with probability R not one of the (n_0 - j) remsining sections will be damaged.

Relationships (4.14), (4.16) are joined to equations (1.1). A variational problem is obtained with breaking right parts (n(t) is a step function) and conditions of isoperimetericness, determining the position of breaks (integrals in (4.16)).

At a large number of sections $(n_0 \gg 1)$ it is possible to construct a continuous approximation of the step law (4.16)

$$\dot{n}=\frac{\lambda}{\ln R}n, \quad n(0)=n_0, \quad n(T)=\text{opt} \qquad (4.17)$$

and therby to free from breaks the right sides of equations and conditions isoperimetericness. Integral characteristics with such approximation are sufficiently exact, starting with $n_0 \approx 10$.

For all stated formulations investigation is made of general properties of optimum control; analytic solutions of model problems are obtained. The problem is formulated of finding the optimum probability R of realization of calculation characteristics (for cargo shipments); examples of its solution are given (V. V. Tokarev, 1962, 1964, 1966).

4.4. Construction of analytic solutions close to optimum. Variational problems appearing in examining problems of optimization lead, as a rule, to complex systems of differential equations. Finding optimum controls and optimum trajectories of motion in analytical form is never or almost never possible. However, analytic solutions are of special interest in connection with their clarity and the possibility of wide parametric analysis.

These indubitable merits served as a base for development of a different series of procedures and methods of detecting of solutions in final form. Of course, these solutions are given at a price of replacement of true gravitational field with a simple one and deviation from the criterion of optimumness of control. The greatest attention is paid to elementary maneuvers with assigned orientation of a vector of rocket acceleration constant in modulus (F. A. Tsander, 1924-1925; A. I. Lur'ye, 1962-1963; V. F. Illarionov and L. M. Shakadov, 1962; M. K. Cheremkhin, 1963; Yu. P. Gus'kov, 1963; G. Ye. Kuzmak and Yu. M. Kopnin, 1963; D. Ye. Okhotsimskiy, 1964; V. V. Beletskiy, 1964; V. A. Yegorov, 1964; N. N. Moiseyev, 1964, 1966; V. V. Larichev and M. V. Reyn, 1965; L. D. Nikolenko, 1965; Yu. G. Yevtushenko, 1966; A. A. Bolonkin, 1965, and others). A detailed survey of these works belonging to a trajectory with a small thrust is contained in a book of G. L. Grodzovskiy, Yu. N. Ivanov and V. V. Tokarev (1966).

4.5. Numerical methods of construction of optimum solutions. As was already noted, in an overwhelming majority of cases investigation of the problem of optimization leads to the necessity of solution of complex variational problems, which is impossible without the use of effective numerical methods. In connection with this in problems of mechanics of flight find wide application existing numerical methods and, on the other hand, during solution of specific problems numerical methods are developed.

Methods of numerical solution of variational problems are divided into direct and indirect methods. The former are based on iterative processes of series decrease (increase) of functional; for application of indirect methods the variational problem is preliminarily reduced to a boundary value problem for a system of differential equations. Let us be limit ourselves to enumeration of those methods which are most often used in problems of flight mechanics:

gradient descent in space of phase coordinates (L. V. Kantorovich, 1945, 1947-1948; B. A. Samokish, 1957; V. A. Brumberg, 1962; Yu. N. Ivanov, 1964; Yu. V. Shalayev, 1964, and others);

gradient descent in space of controls (D. Ye. Okhotsimskiy, 1946; L. I. Shatrovskiy, 1962; T. M. Eneyev, 1963; I. A. Krylov and F. L. Chernous'ko, 1962, and others);

functional method of Newton (L. V. Kantorovich, 1948; G. P. Akilov, 1949; V. A. Vinokurov, 1965; Yu. N. Ivanov, 1965, and others);

finite-dimensional gradient method (I. S. Berezin and N. P. Zhidkov, 1961, and others);

finite-dimensional method of Newton (T. M. Eneyev, A. K. Platonov and R. K. Kazakov, 1960; E. L. Akiy and T. M. Eneyev, 1963; M. K. Gavurin, 1958; V. K. Isayev and V. V. Sonin, 1963, 1965-1966; V. N. Lebedev, 1963; B. N. Rumyantsev, 1963, and others);

methods based on dynamic programming (R. Bellman, 1960; N. N. Moiseyev, 1964-1965; I. M. Sharonov, 1966, and others);

methods based on the principle of optimumness (V. F. Krotov, 1962-1963).

§ 5. Mechanics of Entrance of Spacecrafts into the Atmosphere of a Planet

For a motion of a spacecraft in the period of re-entry into the atmosphere of a planet the appearance of large aerodynamic overloads and heat flow acting on the craft is characteristic.

Therefore, already in the first Soviet works dedicated to prospects of space flight – of S. P. Korolev (1934), F. A. Tsander (see book 1961) and Yu. V. Kondratyuk (1947) – attention was turned to the importance of investigation of this stage of flight, and recommendations were contained on guaranteeing the safe lowering of the craft onto the surface of the planet.

Detailed analysis of trajectories of entrance of spacecrafts into the atmosphere of planets is given in works of V. A. Yaroshevskiy (1964-1965), where is used a dimensionless nonlinear differential second order equation connecting altitude and speed of flight. This equation has the form

$$\frac{d^2y}{dx^2} = -\sqrt{R\lambda}\,\frac{C_y(\bar{V})}{C_x(1)} - \frac{\frac{1}{\bar{V}^2}-1}{y}, \tag{5.1}$$

where $\bar{V} = V/\sqrt{Rg}$ is the ratio of speed to circular speed, C_y is the coefficient of lift, C_x is the drag coefficient corresponding to circular speed $\bar{V} = 1$, R is the radius of the planet, λ is the logarithmic gradient of density,

$$y = \frac{C_x(1)\,S}{2m}\sqrt{\frac{R}{\lambda}}\,\rho,$$

where S and m are the characteristic area and mass of the craft, ρ is the density of the atmosphere, and speed $\bar{V}$ is connected with variable x by the relationship

$$dx = -\frac{C_x(1)}{C_x(\bar{V})}\,\frac{d\bar{V}}{\bar{V}} \tag{5.2}$$

($\bar{V} = e^{-x}$ when C_x = const). The local flight-path angle is determined by the formula

$$\theta = -\frac{1}{\sqrt{R\lambda}}\,\frac{dy}{dx}. \tag{5.3}$$

If speed of entrance into the atmosphere is close to circular, then the solution of this equation in a number of practically interesting cases can be obtained in the form of a series of different structure depending upon the value of the angle of entrance into the atmosphere, the value of the lift-drag ratio, and the law of change of drag coefficient of speed. With the help of such a method are investigated ballistic trajectories of crafts with small lift-drag ratio.

For analysis of trajectories of entrance of crafts with high lift-drag ratio – glide paths and trajectories with reflections – is used the method of averaging nonstationary nonlinear oscillations.

If the speed of entrance into the atmosphere exceeds orbital velocity, then the equation of motion will be converted in such a way as to obtain approximate relationships connecting conditions of entrance into the atmosphere with parameters of trajectory at the point of achievement of minimum altitude during the first descent into the atmosphere. With the help of these relationships is found a simple approximate formula for the width of the corridor of entrance into the atmosphere, valid for crafts with not too low a lift-drag ratio. The reverse problem is examined of finding of the law of change of lift at an assigned dependence of altitude on flight speed. Inasmuch as all these solutions are obtained for dimensionless variables, the results of work are applicable to trajectories of entrance into the atmosphere of different planets.

The trajectories of entrance into the atmosphere of a planet were examined by Yu. M. Kopnin (1967). Introduction of special variables permitted reducing the problem to series determination of the trajectory of satellite in a plane of scanning and projection of the trajectory on the surface of the planet. Approximate solutions are obtained which allow analyzing the influence of values of lift-drag ratio and angle of roll on parameters of yawing motion. We consider the reverse problem of determination of the law of change of angle of roll according to the assigned projection of the space trajectory on the planet surface.

A number of works is dedicated to determination of optimum trajectories of entrance into the atmosphere.

D. Ye. Okhotsimskiy and N. I. Zolotukhina (1964) examined trajectories the lift of which takes alternately maximum negative value and found optimum sequences switchings of lift which allow minimizing maximum overload obtainable on a trajectory of entrance into the atmosphere. A. A. Shilov and Yu. N. Zhelnin (1966) investigated the problem of control of lift during entrance of a craft into the atmosphere and established laws of control optimum in the sense of minimization of maximum overload.

In a work of I. S. Ukolov, Ye. A. Tyulin and E. I. Mitroshin (1967) is examined a scheme for controlling longitudinal distance of the point of landing of the craft during descent into the atmosphere based on analysis of the dimensionless equation of motion. Laws of switching lift are found which ensure a quality of control close to optimum.

In a work of V. S. Vedrov, G. P. Vladychin, A. A. Kondratov, G. L. Romanov and V. M. Shalashkov (1966), and also in a work of V. S. Vedrov, G. I. Vladychin and I. A. Rubtsovoy (1967) are obtained simple laws of control of a craft on the section of descent into the atmosphere ensuring the fastest removal of a winged craft on a landing strip.

G. Ye. Kuzmak and V. A. Yaroshevskiy (1964) examined uncontrolled motion of an exisymmetric-craft around the center of masses during entrance into the atmosphere. With the help of the method of averaging of nonlinear nonstantionary periodic motions is analyzed the influence of initial conditions of angle of attack and angular velocities on amplitudes of oscillations of a craft in dense layers of the atmosphere.

§ 6. Motion of an Uncontrolled Artificial Satellite with Respect to the Center of Masses

A number of geophysical and dynamic problems connected with the study and conquest of outer space requires analysis of the rotation of an artificial object in outer space relative to its center of masses. Without such analysis it is difficult correctly to interpret the readings of instruments on the satellite; motion near the center of masses affects orbit parameters and time of existence of the satellite; there is also a number of other problems requiring knowledge of the orientation of the satellite in space. One should especially note the range of questions connected with the possibility of obtaining a passive orientation of satellites, i.e., orientation caused by the influence of moments of external forces. Essential in these problems is the finding of the natural oriented positions of the satellite and analysis of the stability of these positions and motion in their environment.

Motion of a satellite around the center of masses can be somewhat conditionally divided into two basic types — rotary and librational. In the case of rotary motion the kinetic energy of rotation of a satellite essentially exceeds the work of moments of external forces and the motion of a satellite in a short interval of time close to undisturbed motion corresponding to the absence of moments of external forces. Moments of external forces will put into motion small disturbances, which, however, can be stored with flow of time, leading to essential evolution of motion. If, however, the kinetic energy of rotation of the satellite is low as compared to the work of external forces (or is comparable with it), then motion of the librational type — oscillation of the satellite near a certain oriented direction (radius vector of orbit, vector of magnetic intensity of terrestrial magnetic field, etc.) is possible.

Let us note that during investigation of motion of both types are widely used contemporary methods of investigation — asymptotic methods of the theory of oscillations, theory of stability, numerical methods of analysis, etc.

6.1. Moments of forces acting on a satellite. Investigation of satellite motion around the center of masses usually assumes that the dependence of moments of forces acting on the satellite on its position and speed of rotation is known. These moments in general depend on a complex manner on satellite configuration, distribution of masses, properties of the material of which the satellite is made and the physical properties of the space around the satellite. Therefore, calculation of moments of forces is an independent, sufficiently complex problem. This problem is given much attention in works of V. V. Beletskiy (1958-1959, 1963, 1965), G. N. Duboshin (1958), A. A. Karymov (1962), A. I. Lur'ye (1962-1963), V. A. Sarychev (1961), F. L. Chernous'ko (1965-1966) and others. In these works model formulas approximating exact expressions of moments of forces are offered or exact formulas for concrete configurations of the satellite are calculated.

6.2. Equations of motion. During investigation of librational motion are usually used linear or nonlinear equations of oscillations in a system of coordinates selected in a suitable manner. As variables are most frequently used angles of the type of angles of pitch, bank and yaw. Equations in direction cosines frequently turn out to be convenient for investigation of questions of stability.

It turned out to be a fruitful idea to use as variable components the vector of angular momentum along the fixed axes and the angles of Euler in a system connected with the vector of angular momentum. The equations of motion of a solid in these variables for the first time were offered, apparently, by B. V. Bulgakov (1955), but were developed and found concrete application only with the appearance of problems of the motion of artificial satellites (V. V. Beletskiy, 1958, 1961, 1963, 1965; F. L. Chernous'ko, 1963, and others). These equations are convenient for investigation by asymptotic methods and in different forms and modifications are used for analysis of rotary motion. Also used are other forms of equations; for example, in problems connected with numerical finding of motion, Rodrigues-Hamilton parameters are sometimes used.

6.3. Stabilization and librational motion of a satellite in a gravitational field of forces. Equations of motion of a satellite in a gravitational field in circular orbit allow a particular solution – relative equilibrium in an orbital system of coordinates. In this state of motion the main central axes of inertia of the satellite coincide with the raidus-vector of the orbit tangent to orbit and the normal to the plane of the orbit.

In view of the importance of this solution for creation of systems of passive gravitational stabilization (D. Ye. Okhotsimskiy, 1963; V. A. Sarychev, 1963), the analysis of motion in an environment of relative equilibrium is the subject of many works.

V. V. Beletskiy (1959) has proven that for stability of relative equilibrium in circular orbit it is sufficiently that in relative equilibrium the major axis of the ellipsoid of inertia of the satellite is directed according to the radius-vector of orbit, the minor axis of the ellipsoid of energy according to the normal to the plane of orbit and, consequently, the average axis according to the tangent to the orbit.

The problem of satellite motion around the center of masses is usually considered in a limited formulation: it is considered that motion around the center of masses does not affect the orbit of the satellite. In a limited problem the equations of satellite motion in the gravitational field allow the first intergal – an integral of the Yakbi type which exists only in circular orbit and can be recorded in the following form (V. V. Beletskiy, 1959);

$$A\bar{p}^2+B\bar{q}^2+C\bar{r}^2+3\omega^2\,[(A-C)\,\gamma^2+(B-C)\,\gamma'^2]+ \\ +\omega^2\,[(B-A)\,\beta^2+(B-C)\,\beta''^2]=h_0. \tag{6.1}$$

Here ω is the angular velocity of the motion of the center of masses of the satellite; $\bar{p}$, $\bar{q}$, and $\bar{r}$ are components of the relative angular velocity of the satellite along its main central axes $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$, which correspond to the main central moments of inertia, A. B, and C:

$$\gamma=(i\cdot e_r),\quad \gamma'=(j\cdot e_r),\quad \beta=(i\cdot n),\quad \beta'=(k\cdot n),$$

where e_r is a unit vector in the direction of the radius-vector of the orbit, and n is a unit vector of the normal to the plane of the orbit. In a position of relative equilibrium $j \parallel n$, $k \parallel e_r$, $\bar{p}=\bar{q}=\bar{r}=\gamma=$ $=\gamma'=\beta=\beta'=0$. This motion, as follows from (6.1), is stable if

$$B>A>C, \tag{6.2}$$

what exactly gives the above-indicated stable distribution of the main central axes of inertia of the satellite.

Analysis of small space oscillations of a satellite in circular orbit (V. V. Beletskiy, 1959) showed that, besides the shown sufficient condition of stability, there is a region of values of moments of inertia, in which necessary conditions of stability are fulfilled (motion is stable in linear approximation). In this region the ellipsoid of inertia is close to an oblate spheroid located in relative equilibrium by its least axis according to the tangent, and by the biggest axis according to the normal to the plane of the orbit; average axis, close in value to the biggest axis, is located according to the radius-vector of the orbit.

In elliptic orbit relative equilibrium does not exist, but an analogous important role is played by stable periodical oscillations near the direction of the radius-vector of the orbit. In special detail are investigated oscillations in the plane of the orbit described by the equation (V. V. Beletskiy, 1959)

$$(1+e\cos v)\delta''-2e\sin v\delta'+n^2\sin\delta=4e\sin v \tag{6.3}$$
$$\left(\delta=2\theta,\quad n^2=\frac{3(A-C)}{B}\right),$$

where $\cos\theta = k\cdot e_r$; an independent variable is true anomaly v (e is orbit eccentricity). Application of asymptotic methods to analysis of this equation permits obtaining integral curves describing motion in an amplitude-phase plane (F. L. Chernous'ko, 1963; V. V. Beletskiy, 1965), playing in analysis of nonautonomous oscillations a role analogous to the role of a phase plane for a conservative autonomous system. For example, at small e the solution of equation (6.3) in the neighborhood of the main resonance is in quasi-harmonic form:

$$\delta = a(\nu)\cos[\nu + \varkappa(\nu)], \tag{6.4}$$

where the variables of amplitude $a(\nu)$ and phase $\varkappa(\nu)$ of oscillations are connected by integral

$$\frac{4e}{n+1} a \sin\varkappa + n\left[\frac{a^2}{4} - J_0(a) + 1\right] - \frac{a^2}{2} = \mathrm{const}, \tag{6.5}$$

which allows investigating motion in an amplitude-phase plane $(a, \varkappa)$. In (6.5) $J_0(a)$ is a Bessel function of zero order. The 2π-periodical solutions of equation (6.3) correspond to stationary points of integral curves (6.5), in which

$$\delta = \pm a \sin\nu, \quad n^2 \approx \frac{a \mp 4e}{2J_1(a)}, \tag{6.6}$$

where $J_1(a)$ is a Bessel function of the first kind of the first order. The 2π-periodical solutions will be one (stable) or three (two of them stable, one unstable), if

$$D \equiv e - \left(\frac{2}{3}\right)^{3/2} \frac{(n^2-1)^{3/2}}{2n} > 0 \quad \text{or} \quad D < 0 \tag{6.7}$$

(V. V. Beletskiy, 1965). F. L. Chernous'ko (1963) examined an asymptotic solution of equation (6.3) both at low e, and at any e, but low n^2. In the latter case is revealed, in particular, replacement of the stability of a 2π-periodical solution at $e > 0{,}682$, when the solution responding to $n^2 < 0$ becomes stable; moreover, in the perigee the axis of the least moment of inertia is directed according to the tangent to the orbit. If however, $e < 0{,}682$, then the periodical solution with which in the perigee the axis of the motion of inertia is directed according to the radius-vector $(n^2 \geq 0)$ is stable.

In general analysis of periodical solutions of the equation of plans oscillations in elliptic orbit was made by numerical methods, which permitted obtaining a complete picture of regions of existence of stable periodical oscillations at any value of eccentricity of elliptic orbit and any moments of inertia of satellite (V. A. Zlatoustov, 1964; D. Ye. Okhotsimskiy, 1964; V. A. Sarychev, 1964; A. P. Torzhevskiy, 1964). Investigation of the equation of plane oscillations in elliptic orbit is the subject also of a work of V. V. Beletskiy (1963), I. D. Killya (1963-1964) and A. P. Torzhevskiy (1964).

If the satellite possesses dynamic symmetry, then in circular orbit exist such motions (regular precessions with respect to the normal to the plane of orbit), when the axis of symmetry remains fixed in a rotating orbital system of coordinates. The axis of symmetry is normal either to the radius-vector of the orbit or to the velocity vector and composes a constant (in particular, zero) angle with the normal to the plane of orbit.

These stationary motions, discovered by G. N. Duboshin (1959-1960) and V. T. Kondurar' (1959), were investigated for stability by F. L. Chernous'ko (1954), who found a region of sufficient conditions of stability and a region of necessary conditions of stability. A. P. Markeyev (1965, 1967), using the results of A. N. Kolmogorov and V. I. Arnol'da about stability of motion in canonical systems showed that in the region of necessary conditions of stability motion is stable everywhere, except, perhaps, a set of value of parameter having zero measure.

Application of asymptotic and numerical methods permitted A. P. Markeyev to investigate in detail motion in the vicinity of shown stationary motions both in circular and in elliptic orbits. V. A. Sarychev (1965) obtained conditions of asymptotic stability of stationary motions of a symmetric statellite equipped with a damping device.

6.4. Stabilization and librational motion of a satellite under the impact of moments of forces of a nongravitational nature. Aerodynamic forces can either perturb gravitational stabilization or to promote it. Of fundamental interest is purely aerodynamic stabilization according to the velocity vector of the center of masses of the satellite. The moments of forces of light pressure can stabilize the satellite with respect to the direction to the sun, and moments of magnetic forces with respect to the vector of the magnetic field strength of earth. Also of interest is the question of magnetic disturbances of gravitational stabilization, joint influence of moments of forces of light pressure and gravitational forces, etc. Librational motion under the impact of moment of forces of a nongravitational nature was studied by O. V. Gurko and L. I. Siabkiy (1963), A. A. Karymov (1962, 1964), V. A. Sarychev (1964), V. V. Beletskiy (1965), A. A. Khentov (1967) and others.

6.5. Rotary motion. If the moments of acting forces have a force function, then the first approximation to the motion (in the asymptotic sense) is obtained by averaging the force function according to undisturbed motion, and also, perhaps, according to the orbital motion of the satellite. The satellite accomplishes undisturbed Euler-Poinsot motion relative to an angular momentum vector constant in magnitude; motion of the angular momentum vector itself is described by two canonical equations (V. V. Beletskiy, 1963);

$$\frac{d\rho}{d\nu} = -\frac{1}{L_0 \sin\rho}\frac{\partial \bar{U}}{\partial \sigma}, \quad \frac{d\sigma}{\partial \nu} = \frac{1}{L_0 \sin\rho}\frac{\partial \bar{U}}{\partial \rho}. \qquad (6.8)$$

Here L_0 is the modulus of the vector of angular momentum, constant in the examined approximation; ρ, and σ are two angles determining the position of the vector L of angular momentum in fixed system of coordinates; $\bar{U} = \bar{U}(\rho, \sigma)$ is the average value of the force function. From (6.8) it is obvious that trajectories of the vector of angular momentum are determined by the integral of equations (6.8)

$$\bar{U}(\rho, \sigma) = \text{const.} \qquad (6.9)$$

If the force function is averaged only for Euler motion, then in equations (6.8) $\bar{U}$ depends still on true anomaly v, integral (6.9) does not exist, but motion L is determined by (6.8) in more exact form, where for a number of important cases equations (6.8) with $U = U(\rho, \sigma, v)$ can be accurately integrated (V. V. Beletskiy, 1963, 1965). Thus, for example, in case of gravitational disturbances the vector of angular momentum accomplishes slow precession around the normal to the plane of the orbit at almost constant angular distance from it (V. V. Beletskiy, 1958, 1963, 1965; F. L. Chernous'ko, 1963). If the satellite possesses dynamic symmetry, then

$$\rho = \rho_0, \quad \frac{d\sigma}{dv} = \frac{3}{2}\omega_0 \frac{(A-C)}{L_0}\left(1 - \frac{3}{2}\sin^2\vartheta\right)\cos\rho_0, \tag{6.10}$$

where here ρ is the angle between L and n, σ is the angle of rotation L around n, ω_0 is the frequency of revolution of the center of masses of the satellite along the orbit, $\vartheta = \vartheta_0$ is the constant nutation angle (between L and the axis of symmetry k of the satellite). With respect to the L the dynamically symmetrical satellite accomplishes regular precession.

The described picture of motion corresponds only to nonresonance cases. If however, between characteristic frequencies of motion is a relationship close to resonance, then the picture is complicated and in the first approximation appear disturbances in the motion of the vector of angular momentum, in the magnitude this vector and in motion with respect to the vector of angular momentum, as discovered by A. P. Torzhevskiy (1967) for the case of gravitational distrubances. For example, in the case of fast rotation of a body with a triaxial ellipsoid of inertia during commensurability of the two main Euler frequencies of undisturbed motion, it turns out that the vector of angular momentum L precesses around the normal to the plane of the orbit (analogously the nonresonance case) and, furthermore, accomplishes nutational oscillations (by angle ρ) relative to the normal to the plane of the orbit; during these oscillations L and ρ change so that

$$L \cos \rho = \text{const.} \tag{6.11}$$

V. V. Beletskiy (1958, 1961, 1963, 1965, 1967) investigated rotary motion of a satellite under separate and joint influence of forces of a different nature – gravitational, aerodynamic, magnetic, light pressure, dissipative forces (aerodynamic friction, eddy currents in the shell of the satellite); the influence of the variability of the orbit of the satellite and other factors was examined.

Let us note certain main effects of rotary motion, making the simplest assumptions about the structure of perturbing moments for a dynamically summetric satellite. Aerodynamic disturbances cause precession L at a constant angular distance ϑ from the direction parallel to the vector V_π of the speed of the center of masses of the satellite in the perigee of the orbit. The speed of precession shown is

$$\frac{d\lambda}{dv} = \frac{\rho_\pi c_\pi}{2L_0} aJ_1 \cos\vartheta \quad \left(J_1 = \frac{1}{2\pi}\int_0^{2\pi} \frac{\rho}{\rho_\pi} \frac{(e+\cos v)\sqrt{1+e^2+2e\cos v}}{(1+e\cos v)^2}\, dv\right). \tag{6.12}$$

Here $c_\pi = V_\pi R_\pi$ is the constant of areas of orbital motion (R_π is the perigean radius of the orbit), ρ is the current density of the atmosphere, ρ_π is the density of the atmosphere in the perigee of the orbit, $a = \alpha S l$ is the coefficient of aerodynamic moment proportional to the product of the characteristic area of the satellite S and its characteristic dimension l.

Through the action of constant axial magnetic moment I_0 of the satellite, vector L precesses with a speed of $d\lambda_\kappa/dv$ at a constant angular distance $\varkappa$ from the pole, the direction to which is the angle ρ_1 with the direction of the axis of earth and is contained in a plane normal to the nodal line. Given this,

$$\operatorname{ctg}\rho_1 = \frac{3\sin^2 i - 2}{3\sin i \cos i}, \quad \frac{d\lambda_\kappa}{dv} = \frac{I_0\mu_E}{2pc_\pi L_0}\cos\vartheta\sqrt{1+3\cos^2 i}, \tag{6.13}$$

where p is the focal parameter of the orbit, i is the orbital inclination to the equator, μ_E is the magnetic moment of the magnetic field of earth (assumed dipole and coaxial with the axis of earth). Motion relative to vector L is not destroyed. In particular, in (6.12)-(6.13) nutation angle $\vartheta = \vartheta_0$ is constant.

Analysis of experimental data has shown that effects of type (6.10) and (6.12)-(6.13) dominated for the third Soviet satellite (V. V. Beletskiy, 1961, 1965; Yu. V. Zonov, 1961) and the "Electron-2 satellite (E. K. Lavrovskiy, 1967; S. I. Trushin, 1967) and others.

During nonconservative influences is destroyed also motion with respect to L. Thus, panels of solar batteries skew-symmetrically installed on the satellite create in a flow of rarefied gas a propelling moment untwisting the satellite and causing a number of other effects. The model of such motion on the average possesses the property of preserving values

$$L \sin \vartheta (\operatorname{ctg} \vartheta)^{\alpha} = l_0, \qquad (\operatorname{tg} 2\vartheta)^{2\alpha} \cos \vartheta \sin^2 \vartheta_a = c_0. \tag{6.14}$$

The constant coefficient $\alpha < 0.5$ depends on the aerodynamic properties of the satellite. Integral curves (6.14) permit showing that the satellite can (in a long period) sharply change the state of motion, emerging from conditions of twist $(\vartheta \approx 0)$ to conditions of tumble $(\vartheta \approx \pi/2)$ and back. Together with this is changed angle ϑ_a between vector L and direction V_π and the value of modulus L oscillates. Such effects, as analysis of experimental data shows, dominate in the motion of satellites of the "Proton" type (V. V. Beletskiy, 1967; V. V. Golubkov, 1967; I. G. Khatskevich, 1967).

Dissipative effects along with damping of angular velocities lead as a rule, in the limit to rotation of the satellite around the axis of the greatest moment of inertia (so that the stretched satellite is overturned, and the compressed one is stabilized).

The influence of triaxiality of the ellipsoid of inertia of the satellite on its rotary nonresonance motion is investigated for gravitational moments by F. L. Chernous'ko (1963) and for aerodynamic moments by Yu. G. Yevtushenko (1964-1965) (by averaging of equations on Euler motion on the basis of a scheme proposed by F. L. Chernous'ko).

A number of effects of rotary motion was revealed by Yu. V. Zonov (1959) (eddy currents) and A. A. Karymov (1962, 1964) (ligh pressure).

A description of certain basic effects of rotary motion of the satellite is contained in a book of A. I. Lur'ye (1962) and for L. I. Sedov (1958).

6.6. The influence of moments of forces of internal nature on satellite motion. N. N. Kolesnikov (1962) showed that conditions of stability of relative equilibrium of a satellite as a solid body preserve their form also for a satellite having a cavity wholly filled with viscous liquid. He examined certain satellite problems in the presence of gyrostatic moment (196` 1966).

F. L. Chernous'ko (1965-1966) investigated in detail the problem of motion of a body with a cavity filled with viscous liquid, at low or, conversely, at high Reynolds numbers. This problem was examined also in works of B. N. Rumyantsev (1964), P. S. Krasnoshchekov (1963) and others. Such an investigation may also be applicable to analysis of satellite motion.

Let us assume that, for example, a satellite contains a spheric cavity of radius a filled with viscous liquid with density ρ_0 and viscosity ν, and moment of external forces is absent. Then at low Reynolds numbers Euler equations of satellite motion have the form (F. L. Chernous'ko, 1965)

$$A\frac{dp}{dt}+(C-B)qr=$$
$$=\frac{P_0\rho_0}{\nu ABC}p\left[C(A-C)(A+C-B)r^2+B(A-B)(A+B-C)q^2\right], \qquad (6.15)$$

where two other equations are obtained by cyclic permutation of the letters A, B, C, p, q, r (here $p_0 = \frac{8}{525}\pi a^7$). By virtue of equations (6.15) the value L of the vector of angular momentum is preserved, and kinetic energy T monotonically decreases. The satellite tends toward rotation around the axis of maximum moment of inertia. In the case of a dynamically symmetric satellite the characteristic time of the transition process is

$$T_0 \approx \frac{\nu A^2 C}{\rho_0 p_0 L^2 |A-C|}. \tag{6.16}$$

V. N. Borovenko (1965) and B. A. Smol'nikov (1966) examined the influence on satellite motion of the rotating bodies in it. The latter examined motion in Euler angles relative to the total constant vector of angular momentum L of the body and flywheels; the trajectory of the total vector of angular momentum relative to the main central axes of inertia of the body is given by the integral of energy of motion

$$\begin{aligned}\left[\frac{1}{C} - \left(\frac{\sin^2\varphi}{A} + \frac{\cos^2\varphi}{B}\right)\right] L \sin^2\vartheta + \\ + 2R\left[\sin\nu_0 \sin\vartheta \cos(\varphi - \mu_0) + \cos\nu_0 \cos\vartheta\right] = \text{const},\end{aligned} \tag{6.17}$$

where φ and ϑ are Euler angles of normal rotation and nutation, but ν_0 and μ_0 are angles determining the position in the body of angular momentum of flywheels constant with respect to the body, and R is determined by the value of this angular momentum.

Use of rotating masses for introduction of a satellite to conditions of passive stabilization (V. A. Sarychev, 1967) has essential applied value. Analysis of the influence of elasticity of construction of a satellite on its motion is the subject of a work of V. I. Popov (1965), V. Yu. Rutkovskiy (1965), and T. V. Kharitonovoy (1966).

6.7. On interconnection of translational and rotary motions. Motion with respecto to the center of masses and motion of the center of masses itself, generally speaking are interconnected. It is clear, for example, that the orbit of a nonspherical satellite depends due to atmospheric drag on the orientation of the satellite; on the other hand, change of orientation depends on orbit parameters. Certain questions of interconnection of rotary and orbital motion of a satellite in atmosphere were examined by Yu. G. Yevtushenko (1965); V. V. Beletskiy (1965) defined the idea of effective drag coefficient, which permits taking into consideration the influence of evolution of rotation of the satellite on its orbital motion.

An especially large number of works is dedicated to analysis of interconnection of translational and rotary motion of a satellite in a gravitational field. Here one should note a large cycle of works of V. T. Kondurar' (1959-1963) and works of G. N. Duboshin (1958-1960), in which, in particular, are given general equations of translational-rotational motion of many gravitating bodies. A number of effects of interconnection of translational and rotary motion in a gravitational field is examined in the works of V. V. Beletskiy (1959, 1963, 1965), M. S. Volkov (1962-1963) and others.

6.8. Determination of actual orientation of satellites according to measurements. Readings of various sensors on the satellite permit obtaining information about actual satellite orientation and about actual moments of forces acting on it. For this purpose readings of magnetometers are used, sun sensors, and sensors of angular velocities; and ion traps; data on modulation of radio signal, etc.

Since measurements are taken with certain errors, the natural approach to determination of orientation is satistical treatment of measurements. If at a fixed instant a sufficient quantity of various measurements is necessary, then this permits determining orientation by the local method without knowing anything beforehand about the motion of the satellite near the center of the masses. But usually a sufficient quantity of measurements is dispersed over a considerable interval of time. In this case orientation can be determined only by the integral method, using the whole sum of information for construction of any model of motion. In connection with this the role of models of motion of a satellite near the center of masses is important. As such a model it is possible to take undisturbed motion, differential equations of motion, etc. Algorithms of statistical data processing are usually iterative. Therefore, a large role is played by methods of obtaining zero approximation to satellite motion. This zero approximation is usually obtained from the same information which is later used in statistical treatment. In parallel with determination of orientation is possible determination of moments of forces acting on the satellite. Development of methods of determination of orientation and determination of orientation of a number of Soviet artificial satellites are the subjects of a work of V. V. Beletskiy (1961, 1965, 1967), V. N. Borovenko (1967), Yu. V. Zonov (1961), V. V. Golubkov (1967), I. G. Khatskevich (1967) and others, among which we will note works dedicated to determination of certain parameters of rotation and orientation of satellites according to optical observations for change of their brightness (V. M. Grigorevskiy, 1961, 1963).

6.9. Books and monographs. To questions of motion of a satellite around the center of masses is dedicated a book of V. V. Beletskiy which appeared in 1965. Individual problems of dynamics of satellite rotation are examined also in books of K. G. Bebenin (1964), A. I. Lur'ye (1962), N. N. Moiseyev and V. V. Rumyantsev (1965).

6.10. Remark. In the present paragraph no mention is ever made of important questions of dynamics of guided motion of a satellite around the center of masses (or of passive stabilization of satellites, which is the subject of the next paragraph). Some work on these subjects was carried out by K. G. Bebenin (1964), M. Z. Borshchevskiy (1966), E. V. Gaushus (1963), I. V. Iosiovich (1966), V. P. Legostayev (1966), D. Ye. Okhotsimskiy (1963), B. V. Raushenbakh (1960, 1966), V. A. Sarychev (1963-1964, 1967), B. A. Smol'nikov (1964), and Ye. N. Tokar' (1960).

§ 7. Passive Methods of Stabilization of Artificial Satellites

One important trend in the technology of space flights is creation of oriented artificial earth satellites. Solution of this problem permits carrying out scientific experiments requiring orientation in interplanetary space, return to earth of the satellite or holder with the results of these experiments, creation of a system of relay satellites utilized for purposes of global radio communications and television, launching of meteorological and geodetic satellites and others. Depending upon the problem at hand orientation of an artificial satellite can be carried out with the use of active or passive methods.

For active methods of orientation is necessary the presence on a satellite of orientation sensors and action elements ensuring controlling moments and supporting assigned orientation of the satellite in space. Active systems of orientation are used if it is necessary: 1) to ensure very high accuracy of orientation, 2) to counteract large perturbing moments, 3) and to accomplish a complex program of turns around the center of masses of the satellite. Active systems of orientation require considerable consumption of power and (or) working substance and, as a rule, are intended for a comparatively short time of work.

For projects of satellites without complex program maneuvers, with a long time of existence and accuracy of orientation of the order of 1-5 application of passive systems of stabilization of satellites, it is possible to use the properties of gravitational and magnetic fields, the effect of atmospheric drag and light pressure, gyroscopic properties of revolving bodies and others.

7.1. Systems of gravitational stabilization. Of systems using environment properties the most popular are systems of gravitational stabilization of satellites. The principle of stabilization in these systems is based on the following, well-known property of a central Newtonian field of forces: a satellite with unequal main central moments of inertia has in circular orbit four stable positions of equilibrium corresponding to the coincidence of the biggest axis of the ellipsoid of inertia of the satellite with the radius-vector and the least axis with the binormal to the axis.

Strict proof of the stability of positions of satellite equilibrium based on use of the second method of Lyapunov was given by V. V. Beletskiy (1959). In reference to the moon, which is an example of natural gravitational stabilization of a body relative to an attracting center, necessary conditions of stability of equilibrium positions were obtained in classical celestial mechanics by Zh. L. Lagranzh.

During practical realization of these ideas in systems of gravitational stabilization of satellites appear certain difficulties. The first difficulty is connected with the necessity of damping natural satellite oscillations with respect to the position of stable equilibrium. To ensure damping of natural oscillations the satellite is fulfilled in the form of two parts united by nonrigid connection - satellite and stabilizer. Damping is introduced into the system with use of relative mobility of satellite and stabilizer. The construction of the suspension connecting the satellite with the stabilizer is the most complex element of the system of gravitational stabilization.

A second difficulty appears because of ambiguity of the position of stable equilibrium of the satellite. If the satellite after damping of natural oscillations should occupy an assigned stable equilibrium position, and angles and angular velocities of the satellite in initial moment after separation from the last stage of the carrier rocket are too great, then they must be decreased with the help of a system of preliminary calming values excluding transition of the satellite from one stable position of equilibrium to another. Another solution of the problem is to calm down the satellite in any equilibrium position and already after calming to transfer it with the help of a program turn to assigned equilibrium position.

A third difficulty is caused by a low value of gravitational turning points and the necessity in connection with this of taking special steps to decrease different perturbing influences in order to ensure high enough accuracy of satellite orientation.

The first scheme of a system of gravitational stabilization of artificial satellites was proposed in 1956 by D. Ye. Okhotsimskiy (1963). In this scheme to the body of the satellite with the help of a ball joint is joined a stabilizer carried out in the form of two rods identical in length rigidly fastened with each other with equal loads on the ends. The position of the stabilizer with respect to body of the satellite is fixed by centering springs. The parameters of the stabilizer (length of rods, weight, angle of opening between rods) are selected in such a way as to ensure the necessary parameters of the ellipsoid of inertia of the satellite-stabilizer system. Relative motion of satellite and stabilizer is used for introduction in system of linear damping members.

During investigation of the dynamics of this scheme of a system of gravitational stabilization V. A. Sarychev (1961; 1963) obtained the necessary and sufficient conditions of asymptotic stability of the position of equilibrium of the satellite-stabilizer system, investigated forced oscillations of a system in elliptic orbit and examined the possibility of decreasing the duration of the transition process during extinguishing of natural satellite oscillations.

The necessary and sufficient conditions of asymptotic stability of the position of equilibrium of the satellite-stabilizer system are comparatively easily obtained in general form by plotting the Lyapunov function, the role of which is fulfilled by the Hamiltonian function of the system. The only difficulty is connected with the fact that the derivative from the Lyapunov function by virtue of equations of motion is only a fixed, and not a definite function; therefore, the theorem of the second method of Lyapunov cannot be used in this case without additional investigation.

Necessary and sufficient conditions of asymptotic stability of equilibrium position lead to the following requirements:

1) in the equilibrium position the satellite-stabilizer system should be gravitationally stable, i.e., the axis of the least moment of inertia of the whole system should coincide with the radium-vector, and axis of the greatest moment of inertia with the binormal to the orbit.

2) the values of moments of forces of elasticity counteracting the destabilizing gravitational moments have to be limited from below;

3) at no values of the parameters should the derivative from the Hamiltonian function of the system identically turn into zero, which signifies the absence of such solutions, when the satellite-stabilizer system accomplishes oscillations as a solid body without energy dissipation.

Depending upon the construction of the satellite-stabilizer system and the altitude of orbit can appear the necessity of calculation of atmospheric drag. The influence of drag on circular orbit boils down to change of satellite equiligrium and appearance of forced oscillations with respect to angle of roll and yaw. By selection of the scheme (V. A. Sarychev, 1961) it is possible to leave constant the position of equilibrium and to use reducing aerodynamic moments with respect to angles of yaw and pitch for weakening of requirements made on the relationships between the moments of inertia of the system.

Necessary and sufficient conditions of asymptotic stability of natural oscillations of a system in circular orbit, taking into account atmospheric drag, are obtained by plotting the Lyapunov function. The stability of the satellite-stabilizer system in roll as before is determined only by gravitational moment, it becomes possible to ensure (or to strengthen) stability in pitch and yaw by aerodynamic moment.

Due to the rotation of the atmosphere together with the earth with respect to angles of yaw and roll appear forced oscillations with a frequency equal to the frequency of revolution of the center of masses of the system along the orbit. The amplitude of these oscillations is proportional to the angular velocity of rotation of the earth and the sine of the angle of inclination of the orbit.

It is necessary to note that in principle the influence of atmospheric drag on oscillations of the satellite-stabilizer system can be excluded by selection of stabilizer parameters.

During derivation of the equations of motion of a satellite-stabilizer system with a suspension with three degrees of freedom it was clarified that small oscillations of the system by angle of pitch (in the plane of the orbit) do not depend on angles of roll and yaw, when oscillation with respect to angle of roll and yaw are interconnected and do not depend on pitch angle. This fact permitted transferring in construction of the satellite-stabilizer system to a suspension with two degrees of freedom, but then, having become free from the symmetry of a scheme leading to independence of oscillations in the plane of the orbit because of angles of roll and yaw, to suspension with one degree of freedom (V. A. Sarychev, 1964). Asymptotic stability of the equilibrium position of the system with respect to all angular variables can be ensured, in spite of partial dissipation. Decrease of the number of degrees of freedom of the suspension permits considerably simplifying the construction of the satellite-stabilizer system.

In circular orbit in a medium without drag natural oscillation fade with the passage of time and the satellite-stabilizer system transfers to a position of stable equilibrium. In elliptic orbit there is no equilibrium condition. The system accomplishes in the plane of orbit forced (eccentric) oscillations caused by irregularity of rotation of the orbital system of coordinates. The amplitude of eccentricity oscillations is proportional to the value of the eccentricity of the orbit and depends on the inertial characteristics of the system and the coefficients of friction and elasticity (V. A. Sarychev, 1961, 1963). In the absence of friction in the system it is possible so to select the parameters of the stabilizer that in elliptic orbit the amplitude of eccentricity oscillations of the satellite will be equal to zero. In this case the stabilizer plays the role of a dynamic damper of oscillations. Eccentricity oscillations are easily calculated and can be considered during treatment of results of experiments conducted on the satellite.

Errors of the system of gravitational stabilization (V. A. Sarychev, 1961) are determined by errors of system manufacture and external perturbing moments. Appearance of errors of manufacture are caused by the following circumstances:

1) error in determination of center of masses,

2) error in determination of directions of main dynamic axes,

3) error in reckoning zero of moments of elastic forces.

These errors randomly affect system parameters. During investigation of accuracy of satellite stabilization, of the highest interest are deviations of parameters from the selected nominal values which lead to a nonuniform system of differential equations of motion. Slight deviations of parameters not changing the uniform form of equations of motion can only insignificantly change the transition process and the characteristics of steady motion in elliptic orbit.

It is possible comparatively simply to obtain evident analytic dependences connecting accuracy of satellite stabilization with errors of system manufacture. Analysis of these dependences shows that in weakly elliptic orbits static errors are basic. The amplitude of periodic errors determined by ellipticity of orbit has a higher order of smallness.

Basic external perturbing moments, the influence of which must be taken into account in evaluating the accuracy of systems of gravitational stabilization, are caused by atmospheric drag, the magnetic field of earth, ellipticity of orbit, light pressure, etc. The action of these moments leads to disturbance of the position of equilibrium of the system and appearance of forced oscillations, the amplitude of which is determined by the value of disturbances.

The dynamics and accuracy of systems of gravitational stabilization are essentially influenced also by effects connected with nonrigidness of elements of satellite construction (V. I. Popov, 1965; V. Yu. Rutkovskiy, 1965; T. V. Kharitonov, 1966), especially if one considers that the length of a rod with a load at the end utilized to ensure the necessary reducing gravitational moments can reach tens of meters.

Different systems of gravitational stabilization of artificial satellites differ basically in methods of damping of natural oscillations. Damping can be completely passive, semipassive and active.

In purely passive circuits (D. Ye. Okhotsimskiy, 1963; V. A. Sarychev, 1963) with the use of relative motion of nonrigidly connected satellite parts is introduced linear damping. Practical realization of linear damping is possible, for example, in the form of a magnetic damper the action of which is based on the use of eddy currents, a liquid damper, etc.

An example of a semipassive damping circuit (Ye. N. Tokar', 1966; V. A. Sarychev, 1967) is a gyrodamper consisting of a pair of two-axis gyroscopes, the axis of rotation of rotors of which in the equilibrium position of the satellite are located symmetrically with respect to the normal to the plane of the orbit. Natural oscillations of the satellite cause precession of rotors of gyroscopes connected with the damping device, that leads to energy dissipation of the system. In a semipassive circuit comparatively low energy content is expanded only keeping the speed of rotation of the rotors of the gyroscopes constant.

In systems of gravitational stabilization with active damping the extinguishing of natural satellite oscillations relative to the orbital system of coordinates is carried out with help of an active system, including sensitive and action elements, and only turning points are ensured through the properties of the gravitational field of earth.

The theory of systems of gravitational stabilization of artificial satellites is developed in reference to earth as to an attracting center. However, all the basic results (conditions of stability, eccentricity oscillations, duration of transition process, expressed in number of revolutions of the satellite along the orbit and so forth) are preserved for the moon and the planets of the solar system. The difference appears only in evaluating the influence of perturbing moments and calculation of conditions specific for a concrete planet (for example, practical absence of magnetic field for the moon).

7.2. Systems of aerodynamic stabilization. In circular and weakly elliptic orbits in the range of altitudes from 250 to 350 km for orientation of the axis of symmetry of a satellite with respect to incident flow, the direction of which differs little from the direction of the tangent to the orbit, it is possible to use aerodynamic moments. If the satellite is aerodynamically stable, then with the disturbance of normal orientation appear turning points in pitch and yaw tending to combine the longitudinal axis of the satellite with the velocity vector of leading flow. For removal of uncertainty of turn of satellite in roll (around the longitudinal axis) it is possible, for example, to place in the body of the satellite a rotor rotating with constant angular velocity around an axis perpendicular to the axis of symmetry of the satellite. Gyroscopic moments appearing during rotation of the rotor tend to advance the axis of the rotor with respect to the normal to the plane of the orbit.

An example of a satellite with an aerodynamic to be (to be more exact, aerodyroscopic) system of stabilization is the "Kosmos-149," satellite launched 21 March 1967 into an orbit with an altitude of perigee of 248 km and an altitude of apogee of 297 km and an inclination of the plane of the orbit to the plane of the equator of 48^{o} (A. M. Obukhov, 1967; V. K. Mikhaylov, 1967; V. A. Sarychev, 1967; L. V. Sokolov, 1967). To ensure sufficient reducing aerodynamic moments in pitch and yaw, to the satellite on four long thin rods is attached an aerodynamic stabilizer constituting the lateral surface of the frustum of a cone.

Roll stabilization is ensured with the help of two two-axis gyroscopes on the satellite. Their total angular momentum with normal satellite orientation is perpendicular to the plane of the orbit. The location of the gyroscopes are such that with any disturbance of satellite orientation appear reducing gyroscopic moments with respect to yaw and roll. Thus, in the examined circuit the stability of the satellite in pitch is ensured by aerodynamic moment, in roll by gyroscopic moment, and in yaw by the combined action of aerodynamic and gyroscopic moments. A satellite with an aerogyroscopic system of stabilization possesses only a stable position of equilibrium.

Besides stabilization with respect to roll and yaw gyroscopes ensure also damping of nature satellite oscillations. For this the axes of rotors of gyroscopes are united with damping devices and during precision of gyroscopes appearing during any disturbance of normal orientation of the satellite, occurs dissipation of energy of natural oscillations.

The basic disturbance determining the accuracy of the system of satellite stabilization is caused by entrainment of the atmosphere of the rotating earth, ellipticity of orbit and errors of manufacture of construction. Analysis of readings, of scientific equipment (photometers, three-component magnetometer) on the "Kosmos-149" satellite permits affirming that the aerogyroscopic system ensures satellite stabilization with respect to the system of coordinates according to angles of pitch, yaw and roll with a precision of not less than 5^{o}.

The action of solar light pressure is analogous to the action of atmospheric drag. For artificial earth satellites light pressure is a perturbing influence. However, for spacecraft (in particular, for artificial satellites of the sun) moving distantly enough from the earth and planets in almost circular orbits, light pressure can be used for purposes of stabilization of the craft on the sun.

The idea of joint use of the gravitational field of the sun and its light pressure for stabilization of an artificial satellite of the sun has been proposed by O. V. Gurko and L. I. Slabkiy (1963). They described the construction of a spacecraft which allows stabilizing the action of a gravitational field and light pressure and obtaining sufficient restoring moments at distances of up to 4 A.U. from the sun.

Determination of moments of forces of light pressure acting on a body, and analysis of stability of rotation of a geometrically symmetric artificial satellite in a field of forces of light pressure are the topics of works of A. A. Karymov (1962, 1964).

7.3. Spin stabilization. To ensure constant orientation of a certain axis of a satellite in inertial space is frequently used a stabilization system using gyroscopic properties of rotating bodies. Thus, for example, it is known that stationary rotation of a satellite around axes corresponding to the minimum and maximum moments of inertia is stable. In the presence of dissipative moments only stationary rotation around an axis corresponding to the maximum moment of inertia of the satellite remains stable. External moments caused by the gravitational and magnetic fields of earth, atmospheric drag, and light pressure lead to disturbance of the orientation of a satellite stabilized by rotation. For the preservation of constant orientation of a satellite for a long enough interval of time, influence of external moments must be compensated with the help of a special active device, which is switched on if the deflection of the axis of rotation of the satellite from the assigned direction exceeds the permissible value.

The influence of basic external moments (gravitational, aerodynamic, magnetic, light pressure, and dissipative) on the rotation of an axisymmetrical satellite was investigated in great detail by V. V. Beletskiy (1958, 1961, 1963, 1965). The evolution of the rotation of a satellite with a triaxial ellipsoid of inertia under the impact of gravitational moments was examined by F. L. Chernous'ko (1963), and under the impact of aerodynamic moments by Yu. G. Yevtushenko (1965).

The basic deficiency of systems of stabilization of satellites by rotation is the necessity of exact initial exhibition of the axis of symmetry of the satellite in inertial space, precision twist of the satellite around this axis and periodic compensation of influence of external perturbing moments. For execution of these problems are necessary all elements of active systems — orientation sensors and actuators. The advantage of systems of spin stabilization as compared to active systems is essentially lower expenditure of energy.

In systems of spin stabilization may also be used steady-state solutions for an axisymmetrical satellite in circular orbit obtained by G. N. Duboshin (1959-1960) and V. T. Kondurarem (1959). These solutions correspond to the rotation of a satellite with constant angular velocity around the axis of symmetry, keeping constant its position in the orbital system of coordinates. Cases are possible in which the axis of symmetry is perpendicular to

1) the plane of the orbit,

2) the radius-vector,

3) the tangent to the orbit.

In the last two cases the orientation of the axis of symmetry of the satellite in the orbital system of coordinates is determined by the ratio of the moments of inertia of the satellite and the speed of rotation around the axis of symmetry.

Necessary conditions of stability of these steady-state solutions were obtained by G. N. Duboshin (1960), and sufficient conditions of stability by F. L. Chernous'ko (1964). A. P. Markeyev (1965, 1967) on the basis of results of A. N. Kolmogorov, V. I. Arnol'd and Yu. Mozer proved the stability of steady-state solutions for almost all points of the region were only necessary conditions of stability are carried out, investigated, using methods of averaging, nonlinear oscillations of the axis of symmetry of the satellite in the near-resonance region, and examined the possibility of appearance of parametric resonance in elliptic orbits.

In order to ensure asymptotic stability of stationary rotations is necessary a damping mechanism which works only during deflection from stationary rotation. Such an active damping device has been proposed by V. A. Sarychev (1965). He obtained conditions of asymptotic stability of stationary rotations. The damping mechanism can be realized by purely passive means with the use of ideas proposed in systems of gravitational stabilization (V. A. Sarychev, 1964).

7.4. Stabilization with respect to magnetic field. For certain scientific experiments it can be desirable to orient the satellite with respect to the vector of the magnetic field strength of earth. For this on the satellite is rigidly braced a sufficiently strong magnet, interaction of which with the magnetic field of earth leads to appearance of moments tending to combine the axis of the magnet with the vector of the magnetic field strength of earth.

The basic results according to an analysis of the systems of satellite stabilization with respect to the magnetic field without the mechanism of damping of natural oscillations were obtained by V. V. Beletskiy (1963, 1965) and A. A. Khentov (1967). In these works are investigated forced periodic oscillations of a magnet in the magnetic field of earth, is estimated the perturbing influence of the gravitational field of earth and atmospheric drag, and are clarified conditions of appearance of resonance.

The basic deficiency of satellite stabilization with respect to the magnetic field of earth is connected with the complex character of steady motion of a satellite in a magnetic field, instability of the magnetic field, instability of the magnetic field of earth itself and, as a result, the impossibility of achieving a high enough accuracy of satellite orientation. Till now systems of stabilization with respect to a magnetic field were used only as auxiliary systems of preliminary damping decreasing the amplitude of natural satellite oscillations to values which allow using as basic a system of gravitational stabilization.

In the present paragraph are briefly examined works directly dedicated to theories of motion of satellites with passive systems of stabilizations. Besides these works, in the USSR is carried out a large number of studies in adjacent questions. Of them one should note works about oscillations of a satellite (without a system of stabilization) in elliptic orbit (V. V. Beletskiy, 1963; V. A. Zlatoustov, 1964; D. Ye. Okhotsimskiy, 1964; V. A. Sarychev, 1964; A. P. Torzhevskiy, 1964; F. L. Chernous'ko, 1963; I. D. Kill', 1963-1964), investigations about motion of bodies with cavities filled with viscous liquid (N. N. Kolesnikov, 1962; F. L. Chernous'ko, 1965-1966; P. S. Krasnoshchekov, 1963; B. N. Rumyantsev, 1964), works about optimum (minimum consumption of working substance) deceleration of rotation of a satellite in an inertial system of coordinates (B. A. Smol'nikov, 1964; M. Z. Borshchevskiy, 1966; I. V. Ioslovich, 1966-1967), investigations dedicated to detecting all positions of equilibrium of a system of two bodies united by an ideal ball joint, in orbital system of coordinates and derivation of sufficient conditions of stability of these positions of equilibrium (V. A. Sarychev, 1967). These works are examined more specifically in § 6.

At present passive methods have durably entered the arsenal of technical means used for stabilization of artificial satellites. These methods, not requiring expenditures of working substance and either not connected with expenditure of energy at all or requiring minimum expenditures, turn out to be very effective, when it is necessary to maintain a definite orientation of the satellite for a long period of time and accuracy of the order of a few degrees is sufficient. Systems of stabilization based on the use of passive methods usually turn out to be sufficiently easy both absolutely and in parts of weight of the satellite, which is especially essential for small satellites, including satellites intended for carrying out scientific investigations. Passive methods of stabilization are also very effective on satellites with a long time of active existence, utilized for realization of TV transmissions, telephone and radio communications between continents, and on meteorological satellites. Increase of accuracy, output to the range of altitudes from 500 km to daily orbits, simplification and increase of reliability, and use in passive systems of stabilization of certain elements of active systems will lead to further expansion of the field of application of passive methods.

§ 8. Optimum Correction of Flight Paths of Spacecraft

Development of investigations in region of flight control of spacecraft is intimately connected with solution of problems facing contemporary space technology of exact realization of interplanetary trajectories. The required high accuracy of interplanetary flights is determined by the tendency to create crafts able to carry out close approach with a selected celestial body at gigantic distances from earth.

The need to ensure accuracy of realization of space trajectories exceeding by several orders its terrestrial equivalents produced the necessity of creation of additional systems on board a spaceship which allow correcting orbit in the process of flight. A complexity of creation of similar systems is that they can be built only on the basis of elements of ordinary accuracy. Correctional devices have to be included (at least the last time) in points of trajectory in which the influence of errors of the correction system on corrected orbit parameters does not exceed the permissible level. Inasmuch as among correction errors there are power errors, the formulated requirement means that for correction points of low effectiveness of correction have to be used which can be connected with additional expenditures of fuel. Therefore, to decrease the weight of auxiliary spacecraft systems in many cases it is necessary to conduct a thorough investigation of different properties of motion for the purpose of finding optimum solutions during construction of flight control systems of spacecraft. The theory of correction of spacecraft orbits developed in the last decade is one of the divisions of contemporary astrodynamics and the theory of automatic control. The basic problems of the theory of correction of parameters of spacecraft motion are formulated in a work of G. N. Duboshin and D. Ye. Okhotsimskiy (1963).

The complexity of the problem of correction is determined by the need to minimize the value of total fuel consumption or, what is the same for systems with limited exit velocity of stream, the total characteristic speed of correction in the presence of random errors of determination of orbit and knowingly non-Gaussian errors of performance of correcting maneuvers under conditions, in general, dropping effectiveness of correction with the passage of time. Therefore, if the correction is made late enough, a correcting pulse and considerable additional weight on board the spacecraft can be required. Early correction can be more economical; however, insufficient accuracy of determination of orbit parameters by the moment of its fulfillment can lead to insufficient accuracy of correction and to the necessity of its repeated fulfillment.

The problem of correction can be divided in to three independent problems. These problems include 1) the problem of determination of orbits of spacecraft according to optical and radio observations, 2) the problem of detecting the most effective conditions of correction of the obtained orbit, and 3) the problem of the most rational distribution of measurements and correctional acts on trajectory.

The second problem can be solved independently of the first. Regarding the third problem, it can be solved independently of the first two only with certain simplifying assumptions. Leaving aside questions of determination of orbits, we will give a short survey of basic result attained at present in the theory of correction of flight of spacecraft within the bounds of the last two problems.

Initially the problem on correction was regarded as a problem of selection of a change of speed of flight which leads to hit in an assigned instant in an assigned point of space (see, for example, work of K. V. Kholshchevnikov (1965), V. M. Ponomarev (1965) or "Reference book on cosmonautics" (1966) based on foreign materials). Such a formulation of the problem permits using for calculation of the magnitude and direction of the correcting pulse methods of celestial mechanics (K. V. Kholshchevnikov). However, in reality the variety of required corrected parameters is considerably wider, and practically problems boiling down to such a formulation are absent.

In certain works is examined a correcting change of speed causing an assigned change of orbit elements. Such a formulation strongly complicates the problem and hampers investigation of optimum properties of correcting maneuvers.

In 1959 D. Ye. Okhotsimskiy proposed regarding the correction problem as a problem about change of the coordinates of the point of crossing by the spacecraft of the plane of a figure of a planet. The plane of a figure usually means the plane passing through the center of the planet and oriented orthogonally to the velocity vector of approach of the craft with a nonattracting planet. Such a formulation permitted decreasing the number of corrected parameters of trajectory to two when the instant of approach of the spacecraft with the planet is not essential and permitted considerably simplifying analysis of characteristics of corrections.

In connection with small dimensions of corrected deflections as compared to distances between planets, the problem of correction, in the first approximation, can be examined in linear formulation. However, in the problem of correction is always present a nonlinearity connection of corrected parameters of trajectory with characteristics of motion near the planet. The basic source of nonlinearity in this connection is the attraction of the planet, which should be excluded in linear formulation.

In lectures on mechanics of space flights read by D. Ye. Okhotsimskiy at Moscow University in 1961 was given a procedure for eliminating nonlinear influence of attraction of the planet on corrected parameters of trajectory. This procedure was used in a work of E. L. Akim and T. M. Eneyev (1963), and also in a work of A. K. Platonov, A. A. Dashkov and V. N. Kubasov (1965). Exclusion of influence of attraction of the planet is achieved by using, as corrected parameters, a component of the osculating sighting range at point of closest approach with the planet. The osculating sighting range is a small semiaxis of an osculating hyperbola regarded as a vector lying in the plane of planet-centered motion and orthogonal to the speed of the spacecraft at an infinitely great distance from the planet within the bounds of the two-body problem.

Those or other characteristics of approach with the planet can be simply depicted on a similarly built aiming plane (plane of a figure) (R. K. Kazakov, V. G. Kiselev and A. K. Platonov, 1967). Permissible values of change of characteristics of approach with a planet determine in the plane of a figure the region of probable deviations, disregarding the attraction of the planet. The dimensions of this region determine the required accuracy of realization of interplanetary trajectory or the required accuracy of its correction. The characteristics of correction in this case depend on the degree of influence of the pulse change of velocity vector in some point of trajectory of deviations of coordinates in the plane of a figure.

Along with successful selection of corrected parameters of great importance for investigation of correctional properties of interplanetary orbits is simplicity of analytic expressions for the isochronous derivative of parameters of motion along the trajectory. Very simple expressions for isochronous derivatives were obtained by V. I. Charnym (1965) as a result of study of properties of a linearized system of perturbation equations within the bounds of a two-body problem. These investigations were continued by V. G. Khoroshavtsev (1965), who examined the problem of calculation of isochronous derivative parameters of motion of an artificial satellite for the case of long intervals of time of motion, when trajectory is broken up into sections, and also by V. N. Kubasov (1966), who obtained the analytic dependence of the value of the shown derivative on flight time. Obtained analytic expressions for isochronous derivatives permitted considerably simplifying analysis of characteristics of corrections during flights to the moon and planets.

The general properties of correctional maneuvers during interplanetary flights were investigated in a work of A. K. Platonov (196). He examined in linear approximation the characteristics of a correctional maneuver on different sections of flight path to planets. As correctable parameters of trajectory are used the moment and coordinates of the point of intersection by the spacecraft of the plane of a figure of the planet. It is assumed that correction is made by way of instantaneous change of the velocity vector of flight in one or several points of trajectory and that there is complete information about the motion cf the spacecraft. The investigation is made for the purpose of decreasing the value of the total correction pulse.

Minimization of the value of correcting pulse of speed during single-time correction is possible, if the number of corrected parameters is less than three. For example, in case of correction of two coordinates in the plane of a figure, the pulse of minimum value belongs to the plane of optimum correction stretched to the gradients of these coordinates at the point of correction. The pulse oriented along the normal to the plane of optimum correction does not cause in linear approximation a change of coordinates in the plane of a figure. Therefore, such a pulse direction can be called zero-direction. The pulse along the zero-direction changes only the time of flight to the planet without changing the relative position of the spacecraft and the planet during approach.

Effectiveness of correction in a given point of trajectory can be characterized by the influence of the totality of unit pulses on the coordinates in the plane of a figure. If the direction of the correcting speed can be any direction, such a totality is a unit sphere or a unit circle in the plane of optimum correction. In the space of correcting parameters the reflection of such a sphere is the ellipsoid of the influence of unit pulses of correction, for example, the ellipse of the influence in the plane of a figure.

The stretched ellipse of influence indicates irregularity of directions in the plane of a figure from the point of view of correction. Deflections lying close to the direction of the major semiaxis of the ellipse is easier to correct than deflections, in the direction of its minor semiaxis.

In a work of A. K. Platonov (1966) it is shown that with the passage of flight time ellipses of influence aspire to a circle, the radius of which aspires to zero. For example, during flights to Venus and Mars ellipses of influence are turned into circles approximately 15 days before approach with Venus and two months before approach with Mars. The radius of such a circle in every moment with good accuracy is numerically equal to the time remaining before approach with the planet. In the earlier stages of flight the ellipses of influence can differ by considerable stretchability, especially strong in points of degeneration of characteristics of correction.

The orientation of the optimum correcting pulse in space is connected with zero-direction orientation. It is shown that in the general case of flight to planets zero-direction orientation is preserved neither in absolute nor in orbital systems of coordinates, enduring especially sharp change in points of degeneration of correction characteristics. On the last stage of flight zero-direction is closely to the direction to the planet.

Minimization of the value of the correcting pulse is possible also if some parameter of trajectory does not have to be maintained with high accuracy or has to be maintained with an accuracy of up to a period. A. K. Platonov examines an example of minimization of the value of the correcting pulse during correction of three parameters — two coordinates in the plane of a figure and the time for execution of conditions repeated every twenty-four hours of visibility from earth of approach of the spacecraft with the planet.

In this case first of all one should determine the component of the correcting pulse in the plane of reference of optimum correction and intended for correction of coordinates in the plane of a figure. After that one should select the minimum value of change of flight time by a pulse along zero-direction. One should consider that in the general case of a nonorthogonal bench mark of correctable parameters the gradient of time has a projection on the plane of optimum correction of coordinates and, therefore, the correction of coordinates, in general, changes time of arrival. This forced variation of time depends on the value and direction of the correcting pulse in the reference plane, i.e., in the end, on values of corrected coordinates. The strongest forced change of time occurs, if the correcting pulse in the plane of reference is directed along the projection of the time gradient to this plane.

It is necessary to note that if the value of forced variation exceeds the assigned accuracy of correction of time, then it must be taken into consideration during formation of the correctable deflection, since the optimum moment of approach with the planet depending upon the value of forced variation can be displaced to those or other days (in the examined case time will never require correction of more than 12 hours).

It is interesting to clarify the possibility of degeneration of correction characteristics in some point of trajectory. For this purpose in the work is investigated a matrix connecting motion parameters near the planet with the components of the correcting pulse. It is shown that at an angular distance from the point of correction to the place of approach with the planet equal to 180°, the examined matrix degenerates. Not penetrating analytic details, we will examine the geometric meaning of this fact.

At a point with an angular distance of 180° the correcting pulse change only motion parameters in the plane of the trajectory. The pulse directed perpendicularly to the plane of trajectory turns this plane around the direction to the impact point and cannot change in linear formulation coordinates in the plane of a figure for the planet. If the plane of orbits of the planet and spacecraft are coplanar, then correction of deflections along the binormal is impossible. The ellipse of influence in this case degenerates into a line segment oriented along the line of intersection of the plane of a figure and the plane of trajectory of the craft. Any correcting pulse orthogonal to a gradient of such deflection in the plane of a figure does not cause in this case change of coordinates in the plane of a figure – the plane of optimum correction is not determined. Therefore, in the examined point the zero-direction is turned into a zero-plane perpendicular to the shown gradient. In all the remaining points of the trajectory there is only a zero-direction lying in the plane of the trajectory; however, the effectiveness of correction of deflections along the binormal of the trajectory is close to zero in the vicinity of a point with an angular distance of 180°.

The orientation of the plane of optimum correction significantly differs from the one described, if the orbit of the spacecraft and planet do not lie in one plane. In this case lateral deflection during correction is formed by two causes – change of inclination to the plane of the trajectory and change of the instant of approach with the planet. The latter cause is caused by the circumstance that in case of noncoplanarity of the orbits of the planet and craft, change of moment of arrival leads to exit of the planet from the plane of the trajectory of the craft, i.e., to the appearance of a component of displacement directed along the binormal.

It follows from this that in case of noncoplanar orbits of the craft and planet correcting deflections of speed in the plane of the trajectory, changing flight time, act on lateral deflection. In other words, the gradient of lateral deflection should have a component in the plane of the trajectory. Therefore, at the point of correction with angular distance 180° the gradient of lateral deflection becomes coplanar to the plane of trajectory. The plane of optimum correction in this point coincides with the plane of trajectory. Let us note that the gradient of time in this case coincides with the gradient of lateral deflection, i.e., correction of time, regardless of change of lateral coordinate, in this point is impossible.

It is necessary to stress that given considerations indicate the necessity of investigation of spatial motion during calculation of correction. Correct evaluation of the value of correcting pulse can significantly differ from an evaluation obtained from solution of a two-dimensional problem, since basic power expenditures for correction of lateral deflections essentially depend on the degree of noncoplanarity of the orbits of craft and planet.

In a work of A. K. Platonov (1966) is studied the possibility of degeneration of correlation characteristics in the geocentric section of flight.

During flights to the moon and planets motion in the geocentric section of trajectory is close to parabolic. Investigation of the matrix of derivatives utilized during the work of correction on the assumption that motion occurs along a parabolic trajectory shows that the matrix degenerates if the correctional point is in the perigee of the orbit. In this case the effective direction for correction turns out to be the only direction of flight speed and all three gradients of corrected parameters coincide. In the real case, the trajectory differs from parabolic and strict degeneration of correctional properties does not occur. However, the influence of a pulse collinear to the speed of flight considerably exceeds the influence of a pulse orthogonal to the speed of flight. Physically this is explained by the fact that at the beginning of the orbit, near its perigee, the spacecraft possesses high speed and for turn of velocity vector in space is required a large lateral pulse. At the same time a comparatively small pulse directed along the velocity vector can considerably change the energy of geocentric motion, since the change of energy is proportional to the velocity of flight. Therefore, influence on trajectory with the help of a velocity pulse leads basically to a change of those characteristics of motion which are connected with the energy of geocentric motion. In other words, near earth is practically possible correction of only one parameter of trajectory — either deflection in the plane of a figure along a certain direction or time of arrival.

With passage of time of flight the situation changes. Deceleration with removal of the craft from earth facilitates change of direction of motion and leads to the possibility of affecting deflections in the plane of a figure orthogonal to the line of power influence; characteristics of corrections of different deflections are levelled.

The above-described results illustrate the complex character of dependence of power expenditures on moment of correction. The opinion that these expenditures are lower the earlier correction occurs, true for the last section of orbit, is, in general, incorrect. As was shown, on the trajectory are points where correction is considerably hampered or is simply impossible. These points should be avoided in organization of a single-time correction of orbits of spacecraft.

A. K. Platonov examined also properties of correction in the final section of trajectory before approach with the planet. In view of the proximity of the planet and the craft flying to it, their relative motion can be represented in the first approximation as uniform rectilinear motion, and the set of possible trajectories as a bundle of parallel lines.

The plane of optimum correction in this case is a plane perpendicular to the axis of the bundle. The ellipse of influence is a circle the radius of which is equal to the time remaining up to incidence with the plane of a figure. Thus, outside of dependence on the values and mutual location of speeds of planet and spacecraft, the effectiveness of correction at the end of trajectory is determined by the time remaining up to approach with the planet. In other words, effectiveness of correction is identical during flight to the moon and the planets of the solar system if correction is made with the same time remaining before incidence with the plane of a figure. Another conclusion is the possibility of establishing the necessary direction of the engine for correction near the planet by rotation of the craft around the direction to the planet. The work gives simple relationships determining the characteristics of correction in the section of flight near the planet.

Above were described results of investigation of the characteristic of effectiveness of correction of motion of spacecraft in different points of flight paths to planets. If it is taken into account that only single correction of trajectory is possible, then obtained regularities permit comparatively simply selecting the moment of correction ensuring minimum power expenditures. However, in reality one should consider that the engine of the spacecraft allows repeated switching on and off of thrust. In this case in the problem of correction appear additional free parameters which can be used to decrease power expenditures.

In a work of A. K. Platonov, A. A. Dashkov and V. N. Kubasov (1965), and also in a work of A. K. Platonov (1967) was considered the problem of selection of the best mode of multiple ideal correction. Ideal correction usually means correction deprived of errors of forecast of motion and errors of its performance.

In these works this problem was examined for the case of a limited acceleration. It was assumed that control is carried out for the purpose of keeping rated values of certain functionals in trajectory, for example, coordinates in the plane of a figure of the planet, and the minimized functional is the value of total characteristic speed. The problem was solved by a method developed in works of D. Ye. Okhotsimskiy and T. M. Eneyev (1957) of investigation of variation, which in this case permitted conducting an investigation of properties of obtained optimum conditions of control up to the end.

It was shown that optimum direction of controlling acceleration in any moment of time should correspond to the point of the ellipsoid of influence naving maximum projection on a certain constant vector in the space of corrected parameters, depending on the assigned correcting displacement. The engine should be switched on in points of the trajectory for which the projection exceeds a certain given value. A criterion of absence of conditions of multiple switching on of the engine was formulated consisting of the existence of an everywhere convex totality of ellipsoids of influence in any interval of the examined interval of flight time. Optimumness of the pulse character of conditions of correction was also shown.

Multiple ideal pulse correction was investigated in the above works of A. K. Platonov, A. A. Dashkov and V. N. Kubasov (1965) and A. K. Platonov (1967).

In this case a minimizable value is the sum of the moduli of correcting pulses. In spite of the fact that the space of correcting parameters has special, non-Euclidean metrics, for investigation of the laws of multiple correction it is possible to use a procedure analogous to that used earlier. During investigation of single correction was examined a set of correction pulses equivalent from the point of view of optimization. This set formed a unit sphere in space of speeds or a unit circle in the plane of optimum correction. Conversion of the examined figure of equivalent pulses in the space of corrected parameters permitted obtaining a figure of influence of correction pulses on corrected parameters, for example, an ellipse of influence in the plane of a figure.

Likewise from the totality of equivalent pulses of multiple correction can be obtained the figure of influence in the plane of a figure, with the help of which is investigated the influence of different parameters (for example, moment of correction, direction of pulse, etc.) on the total characteristics of correction.

Thus, for example, with double pulse correction, every pair of pulses, the sum of the values of which is equal to one, corresponds in tne plane of a figure to a figure of influence in the form of a parallelogram, consisting of straight lines connecting pairwise points of ellipses of influence corresponding to selected moments of correction. Every point of this parallelogram car be corrected by a unit total pulse. Modifying moments of correction and direction of action of pulses, we will obtain a set of parallelograms filling the space inside the envelope tangent to the set of ellipses of influence all variable with the passage of time of flight. If this envelope has straight sections, then there are deflections requiring double correction.

It follows from this that for construction of the maximum figure of influence of multiple correction it is necessary to roll the given set of ellipsoids of influence of single correction with a straightening plane. The obtained figure determines different the tactics of correction, depending upon the direction of corrected deflection in the space of corrected parameters. The straightened sections of the obtained convex figure correspond to multiple switching on of the engine (double on a ruled surface, triple on a plane, etc.), and sections belonging to the initial set of ellipsoids of influence to a single switching on of the engine. It follows from this that multiple pulse correction can be required only when the envelope of the set of ellipsoids of influence in the examined interval of time of flight is not everywhere convex; only then will straightened sections exist. Let us note that not everywhere is a convex set of ellipsoids of influence possible only in case of nonmonotonic dependence of their characteristics on time. Otherwise there is always an ellipsoid embracing all the remaining ellipsoids of influence.

It follows from this that for every trajectory there is a finite number of fixed moments and directions of pulses for optimum multiple ideal correction of selected corrected parameters. These moments and direction are determined by the points of contact of the straightening plane of the initial nonconvex set of ellipsoids of influence. The maximum number of switchings on of the engine does not exceed the number of corrected parameters.

Thus, solution of the problem of the best conditions of correction contains cases of multiple switching on of the engine even in the absence of control errors. During such correction occurs alternate displacement in the plane of a figure along the most effective directions in such a manner that the total displacement is equal to the assigned displacement. With every switching on of the engine, aiming in the plane of a figure is produced in a new point, i.e., the characteristics of correction are determined from different conditions. Therefore, such correction can be called nonuniform multiple correction, in contrast to the usual case of uniform multiple correction, in which every subsequent correction corrects errors of the preceding correction, and conditions of correction remain constant.

The nonmonotonic character of the dependence of characteristics of ellipses of influence on time of flight can be connected with degeneration of correction characteristics in a certain point of trajectory. In such cases is observed a sharp irregularity of directions in the plane of a figure from the point of view of effectiveness of correction. Single correction in points of degeneration, as a rule, is practically impossible. In such cases nonuniform correction can be powerfully profitable. In contrast to single correction, one of the switchings on of the engine can occur near the point of degeneration of correction characteristics. The latter circumstance is explained by the fact that in this point of trajectory the effectiveness of correction of a certain linear combination of coordinates in the space of correcting parameters can be considerably higher than in the remaining points of the trajectory. Therefore, it is expedient to correct such a component of corrected deflection namely in the examined point, and to correct the remaining component in any other point of trajectory more effective for them. An example of such a situation is a set of influence ellipses given in a work of A. K. Platonov (1966) in the plane of a figure corresponding to flight to Mars. In the same place is built a figure of influence of nonuniform correction. The powerfully optimum correction of a majority of deflections is nonuniform correction with the switching on of the engine at the point of degeneration at the beginning of flight and then on the ninetieth day of flight.

The described results do not depend on the form of the set of figures of influence. In particular, this set can correspond to correcting pulses, direction of which one way or another is fixed in space. In this case, and also in the case when the number of corrected parameters exceeds the number of independent correcting influences in each point of trajectory, application of nonuniform correction can be necessary regardless of considerations of minimization of total speed.

Similar situations were examined in a work of V. N. Kubasov (1966) and in work of A. K. Platonov and Yu. D. Teterin (1966).

In a work of V. N. Kubasov are investigated features of the method of correction of interplanetary trajectories by a pulse directed along the line spacecraft – sun. With such a method of correction the system of orientation of a spacecraft can be rather simple. Single correction by the given method permits independently changing only one parameter of trajectory – by changing the value of the correcting pulse with its direction fixed. Multiple nonuniform correction is necessary for correction of several parameters of trajectory.

In the work it appears that the total possible number of corrected parameters with such "solar" correction cannot exceed four.

It also appears that during "solar" correction of coordinates in the plane of a figure correction of the time of approach with the planet is impossible. The latter circumstance is explained by the fact that during "solar" correction the correcting pulse belongs to the plane of trajectory and therefore the orientation of the plane of trajectory cannot be modified. In view of this with noncoplanar orbits of spacecraft and planet, approach of craft with planet is possible only at that moment of time when the planet passes the node of the orbit of the craft in the plane of the orbit of the planet.

In the work is investigated also the optimum possible strategy during such "solar" correction.

In a work of A. K. Platonov and Yu. D. Teterin are investigated properties of correction of two or three parameters of spacecraft trajectory when the correcting pulse should belong to a certain plane oriented in a certain way in space.

Such limitation can be dictated by conditions of simplicity of spacecraft construction.

Actually, with optimum correction, depending upon corrected errors, the correcting pulse can be directed in space in any manner. This means that for spacecraft should be provided a corresponding system of orientation having at least two degrees of freedom relative to fixed stars. Simpler is a system of orientation having only one degree of freedom and allowing rotation of the spacecraft around a certain axis. The axis of rotation besides can be directed towards any bright star, for example, the sun, and the correcting pulse can be disposed in a plane perpendicular to this direction. Correction in this case can be called two-component correction since there are only two free components of the correcting pulse correcting not more than two independent trajectory parameters.

In the work it is shown that in flights to outer planets, there are sections of trajectory where the characteristics of such single correction of two trajectory parameters differ little from the characteristics of optimum correction.

The need to correct three parameters of trajectory requires the carrying out of double correction with nonuniform conditions of correction — in such a manner so that as a result of two two-component corrections three selected parameters of trajectory took on the rated value. In the work are investigated general properties of such two-component three-parameter double nonuniform correction. Also investigated are special properties of correction of coordinates in the plane of a figure of the planet and the time of flight when the correcting pulse lies in a plane orthogonal to the direction to the sun.

It is shown that in linear approximation in the plane of the first correction is a direction depending only on the selected two moments of correction such that the component pulse of correction along this direction does not change corrected parameters. It is shown also that for the examined solar correction orientation of such zero-direction does not vary during the period of the whole flight and coincides with the orientation of the binormal of the trajectory. This permits formulating simple rules of strategy during such double correction.

General requirements for systems of correction of interplanetary trajectories are examined in a work of A. A. Dashkov (1966). In this work on the basis of analysis of properties of trajectories are determined the basic requirements for accuracy of fulfillment of correction during flight to Mars, Venus and the moon, and are also discussed certain possible schemes of spacecraft orientation during correction. One of the most interesting methods of orientation of a spacecraft near a planet, useful for correction purposes, is described in a work of A. A. Dashkov and V. V. Ivashkin (1965). This method was used during flight of Soviet automatic lunar stations for maneuver near the moon.

The characteristics of correction of flight paths to Mars and Venus were examined also in a work of A. K. Platonov (1966). The characteristics of correction of flight paths to Jupiter are examined in a work of R. K. Kazakova, V. G. Kiselev and A. K. Platonov (1967).

Thus, the problem of selection of the most effective conditions of correction of the obtained orbit can be considered at present sufficiently developed.

Further investigations in this problem have to be directed towards search of reliable and fast algorithms of determination of optimum points of switching on of a correcting engine.

The most interesting and at the same time the least developed problem at present is the problem of optimum distribution of measurements and correctional acts on trajectory.

The formulation of this problem is contained in a number of foreign and domestic works (A. Rozenblyum, 1961; G. N. Duboshin and D. Ye. Okhotsimskiy, 1963).

The complexity of the examined problem consists of the fact that solution of the problem of correction is connected with the necessity of exact determination of actual parameters of motion during flight. In turn the accuracy of determination of the actual orbit depends with a given composition and accuracy of measured parameters on the location of the measuring interval in orbit and its extent. Tightening the phase of measurements leads, as a rule, to displacement of points of correction to a region of smaller effectiveness.

An unpleasant circumstance is the dependence of, in general, of the accuracy of determination of the orbit of the craft in a certain instant of flight on the magnitude, direction and places of application of correcting pulses in the past and in the future.

A priori errors of performance of future corrections are knowingly non-Gaussian in nature, in view of their dependence on the correcting pulse (this circumstance was given attention by M. L. Lidov). Finally, the above-mentioned results show that the optimum points of correction can gravitate to certain fixed points on the trajectory, if nonuniform correction is optimum. In this case subsequent distribution along the trajectory of correctional acts depends on the direction of displacement in the space of corrected parameters and varies with change in forecast values.

The greatest difficulties at present are in the mathematical formulation of the problem of optimum strategy of correction. Such a formulation should, on the one hand, ensure the possibility of solution of the problem, and on the other hand, be sufficiently strict and allow revealing the basic effects and regularities in its solution. This first of all pertains to selection of a criterion of optimumness during the carrying out of correction.

In spite of the fact that in every concrete flight it is necessary to carry out correction not on the average, but "almost surely," with a probability close to one, during the first attempts to solve the described problem as a simpler criterion of necessary power expenditures was examined mathematical expectation by the total characteristic of speed in the presence of random errors of measurements. In such a formulation the problem is examined by V. A. Yaroshevskiy and G. V. Parysheva (1965), who investigated a one-parameter ideal correction. For the case of Gaussian errors of measurements and the absence of errors of adjustment of pulses the problem is solved up to the end. On the assumption that the value of correcting pulses linearly depends on all preceding values of deflections of the corrected coordinate from the nominal value, are obtained expressions determining the minimum mathematical expectation of consumption on correction, and a simple procedure for determination of optimum moments of correction in the plane effectiveness of correction – accuracy of forecast. There is shown the optimum quality of "under correction" – during every correction one should compensate only part of the deflection of the coordinate from the rated value. In the work is also made a comparison of discrete and continuous correction for simple model problems.

Realizing the weakness of the criterion of optimumness of correction in the form of minimum mathematical expectation of total speed and at the same time wishing to preserve simplicity of computations and analysis, the authors propose using as a criterion of optimumness a certain expression composed of mathematical expectations of separate pulses and approximately equal to the maximum value of total speed realized with the assigned probability.

In a work of V. A. Yaroshevskiy and G. V. Parysheva (1966) is examined the problem of correction of altitude and speed in the pericenter of trajectory of a spacecraft approaching a planet of assignment. The optimum number and distribution of pulses is determined for a different character of change of accuracy of determination of trajectory with change of distance to the planet, initial miss and the distance of the last correction. The problem is solved is assuming the presence of errors of adjustment of pulse not depending on its value. The mathemtatical expectation of the total characteristic speed of correction is minimized on the assumption that correcting pulses have a transverse direction (this direction is close to optimum if correction is made at a distances greater than 2-3 planet radii).

In a more general formulation the problem of statistically optimum pulse correction is examined in a work of I. A. Boguslavskiy (1966). In the work is assumed independence of errors of measurements, errors of performance of correction, and also errors of evaluation of miss in a certain point of trajectory with errors of performance of correction in subsequent points of trajectory. The problem is investigated of detecting during the designing of a system of correction of a method of selection of optimum correcting pulses in all points of trajectory fixed for correction besides the last one.

It is assumed that the method of selection of the last pulse maximizes the possibility of spacecraft incidence in assigned region of permissible miss, and the method of selection of preceding pulses minimizes a posteriori estimators of random variable of total characteristic speed of correction; by experiment are considered flight to a selected point of correction, determination according to the results of measurements of vector of miss and application of a pulse in the selected point of correction (with exact measurement of its actual magnitude and direction).

Initially is examined the simpler problem of minimization of aposteriori (in the above sense) mathematical expectation of total characteristic speed of correction, under the condition that the probability of hit of the spacecraft in a fixed region is equal to the maximum value on a set of correcting pulses at the point of last correction. In general form is described a procedure of detecting of the solution of the problem at hand, based on the method of dynamic programming. In the appendix to the work is given a method of selection of optimum pulses of double one-dimensional correction (analogous to the one examined in a work of V. A. Yaroshevskiy and G. V. Parysheva, 1965).

Further is examined the problem of minimization of the maximum value of total speed of correction determined by the assigned (close to zero) level of probability of the fact that the total speed of correction will exceed the maximum value shown. It is shown that with independent errors of observations and performance of correction (independence of errors of determination of miss from errors of realization of pulses in preceding points is not required) for solution of the problem at hand the procedures of dynamic programming are sufficient. In general form is described a procedure for fixed moments of correction. In the appendix to the work is given an example of such a method for the case of one-dimensional double correction with independence of forecast errors from errors of realization of pulses - in this case the procedure is essentially simplified.

For the case of absence of errors of performance of correction is proven the necessity of "under correction" both in case of minimization of the mathematical expectation of total speed and in case of minimization of maximum value of total speed. At the point of the last correction "under correction" is not optimum. It is also shown that with hit of the vector of miss in a certain region the optimum pulse in a given point of trajectory is equal to zero, where the dimensions of the region are less the higher the level of probability of maximum value of total speed and the greater the errors of forecast in subsequent instants. The dimensions of this region are even greater the greater the error of performance of correction. During calculation of errors of realization of pulse "under correction" in the last point becomes optimum.

In the example of the one-dimensional problem examined in the appendix is investigated the influence of the criterion of optimumness of a double one-dimensional correction on a value of "under correction" in different points of trajectory. As criteria of optimumness of correction are examined the mathematical expectation of total characteristic speed, the approximate expression for maximum value of total characteristic speed (close to the expression proposed by V. A. Yaroshevskiy and G. V. Parysheva for the criterion of optimum correction) and, finally, the maximum value of total characteristic speed realized with assigned probability close to one. It is shown that maximum "under correction" is obtained in the case of application of the first of three criteria and that the other two criteria lead to approximately identical values of "under correction," considerably lower in value.

The most severe formulation of the problem of detecting optimum strategy of correction of flight path of spacecraft is contained in works of D. Ye. Okhotsimskiy, V. A. Ryasin and N. N. Chentsov (1967). In a work of V. A. Ryasin (1966) is examined a simple model problem about single one-parameter correction, in an example to which the author was able to demonstrate a general approach to the problem of correction as to the problem of detecting optimum strategy in a game with nature. Namely, into consideration is introduced a space of elementary outcomes unknown to the observer the elements of which are errors of guidance of the spaceship into orbit and errors of tracking its motion having a known probability of distribution. Further into consideration is introduced information space known to the observer the elements of which are selection of totalities of possible measurements of characteristics of motion for different flight paths of the spacecraft. The set of possible moments of correction and correcting pulses is the space of control. It is shown that if measurements are taken in fixed instants and effectiveness of correction varies monotonically, then it is sufficient to examine the subclass of control for which correction is made in any of the instants coinciding with moments of measurements. Of all controls are examined only those for which the value of total speed of correction does not exceed the given value. Further is introduced the idea of strategy, which is defined as a measurable function assigned in information space with values in the space of controls. In other words, every set of measurements along the whole trajectory is coordinated with certain instants and correcting pulses (according to a rule determined by strategy). With every selected strategy is connected a certain set of possible errors with a known probability in the space of elementary outcomes and, consequently, a definite probability of hit in the region of permissible errors in the space of corrected parameters. If this probability is maximum, then the corresponding strategy is optimum. The problem is to establish the existence of optimum strategy and to give a method of its construction.

In the given formulation are absent errors of performance of correction; however, they can be considered expanded by determination of the space of elementary outcomes.

The basic result of author, undoubtedly, beneficially affecting the trend of further investigations, is the idea about the fact that the search for optimum strategy signifies the search of a certain partition of information space into regions corresponding to the carrying out of correction in identical instants, and the construction of optimum functions for determination of the characteristics of the correcting pulse in each point of information space. Moreover, in the work is proved that making certain, very general, assumptions with any partition of information space can be found functions which are optimum functions of the correcting pulse. In other words, it is possible to examine only strategies in which the function of correcting pulse does not depend on the function of the moment of correction, and to detect in this subclass of strategies the strategy with optimum moments of correction.

Optimum functions of correcting pulse in information space can be selected regardless of its partition. The optimum partition of information space depends on the functions of the correcting pulse and can be found with the help of the principle of dynamic programming. In the work is proved that every such region of information space constitutes a cylindrical set with a base in the subspace of measurements preceding the moment of carrying out of correction; the decision about correction is made on the basis of past information and does not depend on future information. The shown regions of information space corresponding to identical moments of correction do not cross, and their sum composes the whole space. Therefore, the procedure of search for optimum partition of information space looks as follows:

1) construction in the whole information space of characteristics of the magnitude and direction of the correcting pulse supplying the maximum probability of fulfillment of the assignment during flight;

2) determination in all of information space of the probability of fulfillment of assignment with the use of the built optimum correcting pulse functions;

3) sequential, starting with the end of flight, separation in information space of sets of elements for which fulfillment of correction in a given moment leads to a greater probability of success than fulfillment of correction in subsequent instants, taking into account new information;

4) partition of information space to unknown regions is as follows: separation of a set for which the carrying out of correction in the first moment is the most optimum; separation then of the crossing of the remaining part of the information space and the set, for which the carrying out of correction in the second moment is more optimum than in later moments; separation of the crossing of the remaining part of information space and the set for which the carrying out of correction in the third moment is more optimum than in the one following it, etc.

The process of making the decision about correction consists of the following. In consecutive instants the observer obtains a selection of measurements. If after the first measurement the observer discovers that this measurement belongs to the base of the cylindrical region of correction in the first instant, then correction is made in the first instant; otherwise a second measurement is taken, and it is checked whether the obtained selection from two measurements belongs to the base of the cylindrical region corresponding to the carrying out of correction in the second instant, etc.

Finding of optimum regions involves great difficulties and is facilitated in the case of normally distributed errors. In the work is given a solution of the problem for the case of a one-parameter ideal correction within the bounds of a model formulation with Gaussian errors of removal and measurements.

In a work of D. Ye. Okhotsimskiy, V. A. Ryasin, and N. N. Chentsov (1967) the described method is used for the case of one-parameter double ideal correction.

Strategy in double correction is determined by the partition of information space into cylindrical sets corresponding to identical moments of carrying out two corrections to which are assigned controlling functions of the first and second correction. A theorem is given of the existence of the optimum strategy of double correction. It is shown that the value of "under correction" tends to zero with increase in accuracy of measurements. With almost exact measurements optimum strategy can be essentially simplified. An example of close to optimum strategy of double correction is given.

Thus, the problem of optimum correction of orbits of spacecraft by the efforts of Soviet scientists is investigated and in considerable degree is advanced on the way to solution. Basic efforts were directed towards detecting optimum conditions of correction, investigation of general properties of correctional maneuvers, selection of convenient corrected parameters, construction of technically simple methods of correction, detecting of approximate criteria of optimumness, which allow solving the problem by simple means, investigation with the help of model problems of basic effects and regularities during optimum imperfect correction, on strict formulation of the problem of optimum imperfect correction and detecting of methods of its solution. Successes of Soviet scientists in the region of practical application of the theory of optimum correction is indicated by the carrying out of corrections of orbits of spacecraft launched by the Soviet Union to the moon and planets of the solar system (see: "Investigation of upper atmosphere and outer space." Report of KOSPAR, 9th plenum, Vienna, 1966).

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ABSTRACT

(U) This document deals with the mechanics of space flight and the new divisions of mechanics to which the progress of space rocket technology gave life.